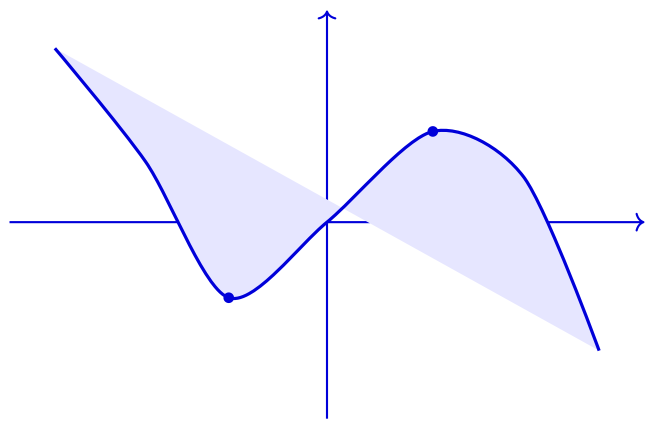
Summit MATH 251: Calculus III

Summit fully illustrated textbook edition



Original Summit-authored instructional text generated from the live course runtime, bibliography layer, and assessment structure.

March 22, 2026

@@TOKEN_0@@ Summit first edition draft @@TOKEN_1@@ college @@TOKEN_2@@ 4 @@TOKEN_3@@ 14 weeks @@TOKEN_4@@ 12.9 hours/week

Originality note

This textbook is a Summit-authored instructional text. It is informed by the course bibliography in @@TOKEN_0@@ and by open academic references used elsewhere in Summit, but it does not copy or restate any single commercial textbook.

How this textbook was built

This book was generated from the live Summit course runtime for Calculus III: the syllabus, lesson sequence, reading chapters, guided practice, homework sets, quizzes, mastery exam, and workload standard. The design goal is to give a student a usable, course-complete book while preserving original Summit wording and sequencing.

An original Summit multivariable calculus course focused on vectors, geometry in space, partial derivatives, multiple integrals, and the first major theorems of vector calculus.

Mathematics chapters should move from concept to representation to fluent execution. Students should always know what the symbols mean before they try to manipulate them.

This volume is structured as a teaching book rather than a bare note pack. Every chapter contains explanation, worked examples, guided practice, chapter homework, and a rear answer key so the student can study independently and still get disciplined feedback.

Course use guide

- Read one chapter at a time in sequence; each chapter is aligned to a live lesson block in the course workspace.
- Rebuild the worked examples before attempting the graded homework or quiz material.
- Keep a scratch notebook beside the text and write down assumptions, diagrams, and the points where you usually get stuck.
- Use the course tutor, guided practice, and homework only after you can explain the chapter in your own words.

Contents

Course map

- 4 live lesson chapters
- 2 graded homework checkpoints
- 2 timed quizzes
- 1 cumulative mastery exam
- 6 declared course outcomes

Prerequisite and readiness position

Course prerequisites: calculus-ii. Readiness clearances: calc-ii-credit.

Summit Calculus III starts after the single-variable sequence is complete. Students must arrive prepared for vectors, multivariable thinking, and heavier spatial reasoning from the first week.

Semester workload standard

Summit models this course as @@TOKEN_0@@ across a 14-week term plus final assessment window. The expected distribution is:

- Contact-equivalent instruction: 56 hours
- Reading: 18 hours
- Practice and problem solving: 62 hours
- Homework: 24 hours
- Lab, design, and reporting: 0 hours
- Exam preparation: 20 hours

Expected volume:

- 170-210 multivariable problems covering vectors, partial derivatives, multiple integrals, and vector-calculus setup.
- 10 graded homework sets totaling 45-55 multistep problems with full written solutions.
- No standalone lab block; explanatory writeups are embedded inside homework, corrections, and exam review.

Reference basis

Primary synthesis anchors from the bibliography for this course (50 listed references total):

1. Calculus
2. Calculus
3. Thomas' Calculus
4. Calculus, Volume 1
5. Active Calculus
6. Calculus for Engineering Students
7. Applied Calculus for Scientists and Engineers
8. Introduction to Integral Calculus

Chapter 1

Chapter 1 Vectors and geometry in space

Chapter purpose

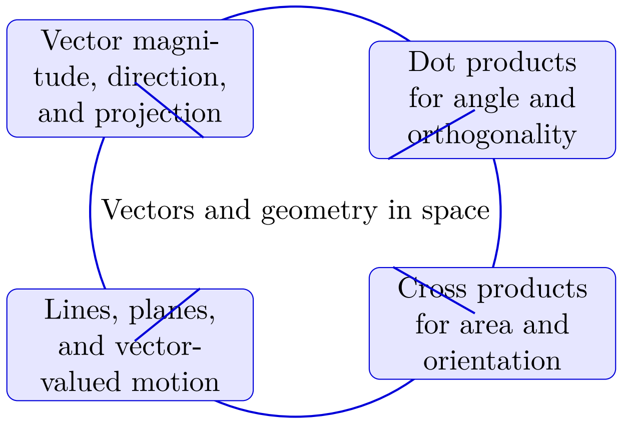
The course starts by extending geometry into three dimensions. Students learn vector algebra, dot and cross products, vector-valued functions, and the equations of lines and planes. The payoff is modeling: once direction, angle, and orientation are clear, students can describe motion and geometry in the language used throughout engineering and physics.

This chapter sits at the opening of Calculus III. It develops Vector magnitude, direction, and projection, Dot products for angle and orthogonality, Cross products for area and orientation, and Lines, planes, and vector-valued motion so that the student can move from explanation to execution without losing the thread of the course.

The central habit in this chapter is to move across words, graphs, formulas, and worked algebra without losing meaning. A correct answer is not enough on its own; the student should be able to explain why the setup is valid and how the result fits the larger mathematical structure of the course.

Core ideas

- Vector magnitude, direction, and projection
- Dot products for angle and orthogonality
- Cross products for area and orientation
- Lines, planes, and vector-valued motion



How to think through this chapter

Problem solving in this family starts with naming the structure of the task. Students should ask which theorem, definition, or representation controls the problem before choosing a computational path. Once the structure is clear, algebraic execution should be clean, annotated, and checked against the expected behavior of the function or model.

When working this chapter, keep the following question active: @@TOKEN_0@@ A good student answer should connect setup, assumptions, and conclusion instead of only chasing a final number or sentence.

Calculus III opens by moving from line-based thinking to space-based thinking. Vectors are the language that keeps magnitude and direction tied together when geometry, motion, and force all matter at once.

Why vectors deserve their own language

A single real number can describe height or temperature, but it cannot describe a push in space. Engineering quantities like velocity and force need both amount and direction, so vectors become unavoidable. This is not extra notation for its own sake. It is the minimum structure needed to describe three-dimensional behavior honestly.

Once students accept that, dot products, cross products, and vector equations stop looking like arbitrary inventions. Each operation extracts a different geometric relationship from directional data.

Coordinates are bookkeeping, not the idea itself

A vector written with components is a convenient coordinate representation, but the vector is not the list of numbers. It is the underlying directed quantity. This distinction matters because the same geometric object can be represented in different frames or bases.

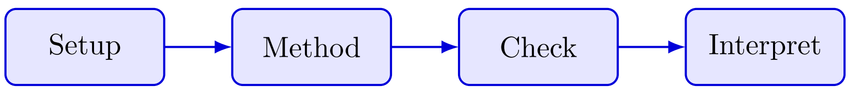
Students who hold onto the geometry tend to make fewer mistakes with projections and planes, because they see what the numbers are meant to encode.

Projection is one of the first genuinely useful vector tricks

Projection answers a subtle but common engineering question: how much of one quantity points in the direction of another? That idea appears in work, signal decomposition, and geometric constraints. It is much more than a classroom exercise.

A helpful mental model is "shadow in a chosen direction." Once students see projection that way, the formulas are easier to remember and easier to rebuild from scratch.

Worked example



@@TOKEN_0@@ Find the equation of the plane through (1,2,0) with normal vector <2,-1,3>.

1. Use the point-normal form n dot <x - x0, y - y0, z - z0> = 0.
2. Substitute the normal vector and point: 2(x - 1) - (y - 2) + 3(z - 0) = 0.
3. Expand and simplify.
4. The plane equation is 2x - y + 3z = 0.

Read this example twice: once for the flow of ideas and once for the technical structure of the solution.

Worked-through guided example

@@TOKEN_0@@ Find the dot product of a = <1, 2, -1> and b = <3, 0, 4>.

1. Compute 1ů3, 2ů0, and (-1)ů4.
2. Add those three products.
3. Interpret the sign of the dot product as positive, zero, or negative alignment.

The dot product is 3 + 0 - 4 = -1, indicating slight opposite-direction alignment overall.

Instructor commentary

Students should annotate this chapter for structure, not just facts. Mark where the argument changes direction, where the method requires a hidden assumption, and where the conclusion becomes more general than the worked example. If the chapter feels easy while you are reading it but difficult when you close the page, you have not yet converted recognition into mastery.

The most effective study pattern is read, annotate, rebuild the worked example without looking, and then solve several short-to-long problems in one sitting so the idea becomes automatic.

Practice while you read

Practice Set: Vectors and geometry in space

Connect component calculations to geometry, direction, and projection.

@@TOKEN_0@@ Find the dot product of a = <1, 2, -1> and b = <3, 0, 4>.

- Hint: Multiply corresponding components and then add the results.
- Step 1: Compute 1ů3, 2ů0, and (-1)ů4.
- Step 2: Add those three products.
- Step 3: Interpret the sign of the dot product as positive, zero, or negative alignment.
- Checkpoint: a ů b = -1

@@TOKEN_0@@ Find the projection of v = <4, 2> onto u = <1, 1>.

- Hint: Use proj_u(v) = ((vůu)/(uůu))u.
- Step 1: Compute v ů u and u ů u.
- Step 2: Form the scalar factor (vůu)/(uůu).
- Step 3: Multiply u by that scalar.
- Checkpoint: Projection = <3, 3>

Chapter homework

@@TOKEN_0@@ Geometric modeling, gradient interpretation, and local linear approximations.

1. Find the angle between vectors <1,2,-1> and <2,0,3>.
2. Find the line of intersection between the planes x + y + z = 3 and x - y + z = 1.
3. For f(x,y) = x e^(xy), find the directional derivative at (1,0) in the direction <3,4>.

4. Find the tangent plane to $z = x^2 + xy + y^2$ at (1,1,3).

Answers for these homework problems appear in the back-of-book answer key.

Chapter summary and study notes

- Recognize whether a problem is fundamentally about projection, orientation, or intersection.
- Move among vector, parametric, and symmetric line descriptions.
- Interpret the cross product geometrically instead of as a memorized determinant pattern.

Study tips

- Sketch the geometry even when the components are given numerically.
- Use the dot product for alignment questions and the cross product for perpendicular area or orientation questions.
- Treat projection as a directional shadow, not a formula to memorize blindly.

Common traps

- Confusing a scalar component with the full projection vector.
- Mixing up magnitude with direction when normalizing.
- Using cross products when the question is really about parallel alignment.

Family-level errors to watch for

- Starting algebra before identifying the governing definition or theorem.
- Dropping notation, units, or sign conventions in the middle of a calculation.
- Treating a symbolic answer as finished without interpreting what it means.

Chapter 2

Chapter 2 Partial derivatives, gradients, and local linear models

Chapter purpose

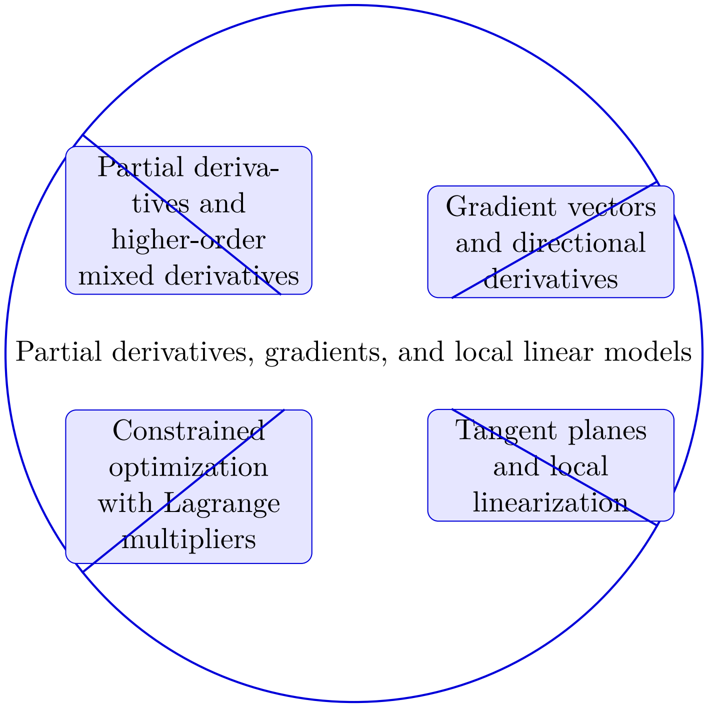
Students now differentiate functions of several variables. The lesson covers partial derivatives, tangent planes, chain rules, directional derivatives, and gradients, then closes with local optimization and Lagrange multipliers. The central habit is interpretation: each partial derivative is a rate with one variable allowed to move while others are frozen.

This chapter sits in the middle of Calculus III. It develops Partial derivatives and higher-order mixed derivatives, Gradient vectors and directional derivatives, Tangent planes and local linearization, and Constrained optimization with Lagrange multipliers so that the student can move from explanation to execution without losing the thread of the course.

The central habit in this chapter is to move across words, graphs, formulas, and worked algebra without losing meaning. A correct answer is not enough on its own; the student should be able to explain why the setup is valid and how the result fits the larger mathematical structure of the course.

Core ideas

- Partial derivatives and higher-order mixed derivatives
- Gradient vectors and directional derivatives
- Tangent planes and local linearization
- Constrained optimization with Lagrange multipliers



How to think through this chapter

Problem solving in this family starts with naming the structure of the task. Students should ask which theorem, definition, or representation controls the problem before choosing a computational path. Once the structure is clear, algebraic execution should be clean, annotated, and checked against the expected behavior of the function or model.

When working this chapter, keep the following question active: @@TOKEN_0@@ A good student answer should connect setup, assumptions, and conclusion instead of only chasing a final number or sentence.

Partial derivatives generalize the derivative by allowing a function to depend on several variables at once. The key idea is disciplined isolation: one variable changes while the others are temporarily held fixed.

Several inputs mean several directions of change

If a quantity depends on x and y, then no single derivative can describe all local behavior. The function may climb quickly in one direction and barely change in another. Partial derivatives measure these directional sensitivities one coordinate direction at a time.

That is why multivariable calculus feels less like extending old formulas and more like learning to describe terrain. The graph is now a surface, and local behavior depends on which way you move across it.

Holding variables fixed is a deliberate modeling move

Students sometimes worry that freezing one variable is artificial. In fact, it is exactly what analysis often requires. To understand how temperature reacts to pressure, you may need to hold volume constant. To understand cost sensitivity, you may vary one design variable while treating the rest as fixed.

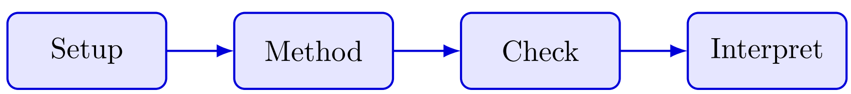
The partial derivative is therefore not a compromise. It is a clean way to isolate influence inside a larger system.

The gradient turns separate derivatives into a local guidance system

Once the partial derivatives are collected into the gradient, the course gains a tool that points in the direction of fastest increase. This is one of the first times students see several separate calculations fuse into a geometric object with direct meaning.

That fusion matters later in optimization and field theory. The gradient is not just a container. It is local directional intelligence.

Worked example



@@TOKEN_0@@ For f(x,y) = x^2 y + e^(xy), find fx and fy.

1. For fx, treat y as constant: fx = 2xy + y e^(xy).
2. For fy, treat x as constant: fy = x^2 + x e^(xy).
3. Each derivative reflects change with respect to one input while the other stays fixed.

Read this example twice: once for the flow of ideas and once for the technical structure of the solution.

Worked-through guided example

@@TOKEN_0@@ Find fx and fy for f(x, y) = x^2y + 3xy^2.

1. For fx, hold y constant and differentiate term by term.
2. For fy, hold x constant and differentiate term by term.
3. Write both partial derivatives cleanly.

Treating y as constant gives fx = 2xy + 3y^2. Treating x as constant gives fy = x^2 + 6xy.

Instructor commentary

Students should annotate this chapter for structure, not just facts. Mark where the argument changes direction, where the method requires a hidden assumption, and where the conclusion becomes more general than the worked example. If the chapter feels easy while you are reading it but difficult when you close the page, you have not yet converted recognition into mastery.

The most effective study pattern is read, annotate, rebuild the worked example without looking, and then solve several short-to-long problems in one sitting so the idea becomes automatic.

Practice while you read

Practice Set: Partial derivatives and gradients

Practice controlled one-variable-at-a-time differentiation and local interpretation.

@@TOKEN_0@@ Find fx and fy for f(x, y) = x^2y + 3xy^2.

- Hint: Differentiate with respect to one variable while treating the other as a constant.
- Step 1: For fx, hold y constant and differentiate term by term.
- Step 2: For fy, hold x constant and differentiate term by term.
- Step 3: Write both partial derivatives cleanly.
- Checkpoint: fx = 2xy + 3y^2, fy = x^2 + 6xy

@@TOKEN_0@@ Find the gradient of f(x, y) = x^2 + y^2 at the point (1, -2).

- Hint: The gradient is built from the first partial derivatives.
- Step 1: Differentiate to get fx and fy.
- Step 2: Evaluate both partial derivatives at (1, -2).
- Step 3: Package the result as a vector.
- Checkpoint: f(1, -2) = <2, -4>

Chapter homework

@@TOKEN_0@@ Geometric modeling, gradient interpretation, and local linear approximations.

1. Find the angle between vectors <1,2,-1> and <2,0,3>.

2. Find the line of intersection between the planes x + y + z = 3 and x - y + z = 1.
3. For f(x,y) = x e^(xy), find the directional derivative at (1,0) in the direction <3,4>.
4. Find the tangent plane to z = x^2 + xy + y^2 at (1,1,3).

Answers for these homework problems appear in the back-of-book answer key.

Chapter summary and study notes

- State clearly which variable is held constant in a partial derivative.
- Use the gradient as the direction of steepest ascent with proper normalization when needed.
- Check candidate points for constrained extrema carefully.

Study tips

- Say out loud which variables are being held fixed before differentiating.
- Check whether symmetry in the formula should produce symmetry in the partial derivatives.
- Interpret the gradient as direction plus steepness, not merely as a list of derivatives.

Common traps

- Differentiating with respect to one variable while accidentally letting another vary too.
- Forgetting that a partial derivative is still a function of all remaining variables.
- Treating the gradient as notation only and missing its geometric meaning.

Family-level errors to watch for

- Starting algebra before identifying the governing definition or theorem.
- Dropping notation, units, or sign conventions in the middle of a calculation.
- Treating a symbolic answer as finished without interpreting what it means.

Chapter 3

Chapter 3 Double integrals, triple integrals, and coordinate changes

Chapter purpose

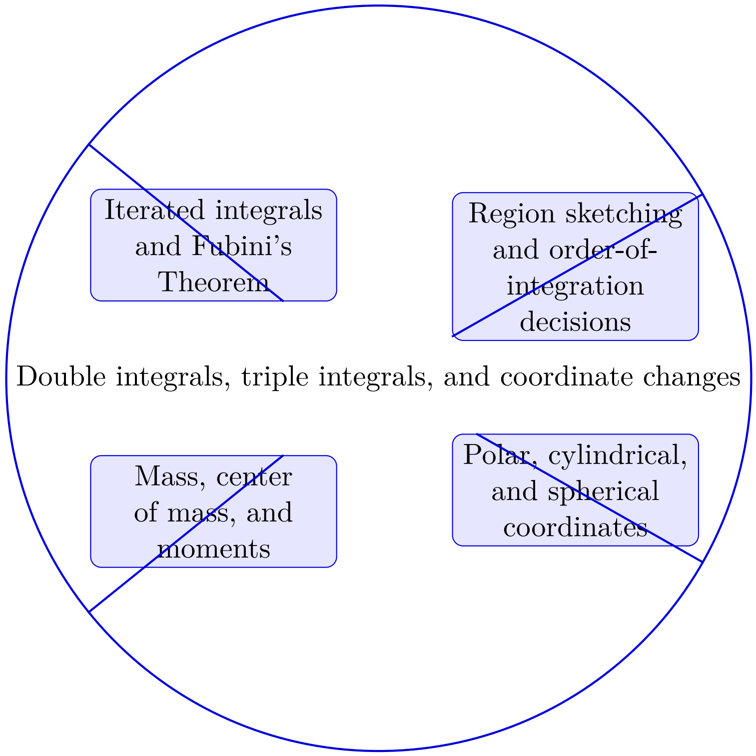
This lesson turns multivariable calculus into accumulation over regions in the plane and in space. Students sketch domains, reverse order of integration when helpful, and use polar, cylindrical, and spherical coordinates where symmetry makes them superior. More than anywhere else in the course, region geometry drives success.

This chapter sits in the middle of Calculus III. It develops Iterated integrals and Fubini's Theorem, Region sketching and order-of-integration decisions, Polar, cylindrical, and spherical coordinates, and Mass, center of mass, and moments so that the student can move from explanation to execution without losing the thread of the course.

The central habit in this chapter is to move across words, graphs, formulas, and worked algebra without losing meaning. A correct answer is not enough on its own; the student should be able to explain why the setup is valid and how the result fits the larger mathematical structure of the course.

Core ideas

- Iterated integrals and Fubini's Theorem
- Region sketching and order-of-integration decisions
- Polar, cylindrical, and spherical coordinates
- Mass, center of mass, and moments



How to think through this chapter

Problem solving in this family starts with naming the structure of the task. Students should ask which theorem, definition, or representation controls the problem before choosing a computational path. Once the structure is clear, algebraic execution should be clean, annotated, and checked against the expected behavior of the function or model.

When working this chapter, keep the following question active: @@TOKEN_0@@ A good student answer should connect setup, assumptions, and conclusion instead of only chasing a final number or sentence.

Multiple integrals extend accumulation into regions and volumes. The real challenge is not symbolic integration alone. It is understanding the geometry of the domain and deciding how to sweep through it cleanly.

Region setup is the heart of the problem

In double and triple integrals, the integrand tells you what is being accumulated, but the bounds tell you where the accumulation happens. Many student errors come from treating the limits as afterthoughts. In reality, the limits are the geometry translated into calculus.

That is why sketches matter so much. A good sketch often reduces a long analytic struggle to an obvious choice of order and boundaries.

Order of integration is a strategic choice

Fubini theorem makes repeated integration possible, but it does not force a single order. One order may create ugly bounds or impossible antiderivatives, while another makes the same region simple. Students should learn to treat order as a design decision.

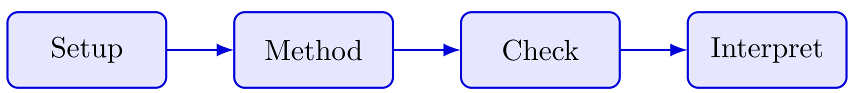
This is a valuable engineering habit beyond mathematics: the problem statement may be fixed, but the representation is often negotiable.

Coordinate changes are a sign of respect for the geometry

Polar, cylindrical, and spherical coordinates appear when rectangular coordinates stop matching the shape of the region. The Jacobian factor can feel technical at first, but its role is honest: it corrects the size of small pieces in the new coordinate system.

A useful way to remember this is that coordinates change the bookkeeping, and the Jacobian keeps the bookkeeping fair.

Worked example



@@TOKEN_0@@ Evaluate double integral over the unit disk of (x^2 + y^2) dA.

1. Use polar coordinates because the integrand becomes r^2 and the region is 0 <= r <= 1, 0 <= theta <= 2pi.
2. Include the Jacobian factor r, giving integral of r^3.
3. Compute integral from 0 to 2pi integral from 0 to 1 of r^3 dr dtheta.
4. The value is pi/2.

Read this example twice: once for the flow of ideas and once for the technical structure of the solution.

Worked-through guided example

@@TOKEN_0@@ Evaluate from x=0 to 1 from y=0 to 2 of (x + y) dy dx.

1. Compute the inner integral 0^2 (x + y) dy.
2. Simplify the resulting function of x.

3. Integrate that expression from 0 to 1.

The inner integral is 2x + 2. Integrating from 0 to 1 gives 0^1 (2x + 2) dx = [x^2 + 2x]0^1 = 3.

Instructor commentary

Students should annotate this chapter for structure, not just facts. Mark where the argument changes direction, where the method requires a hidden assumption, and where the conclusion becomes more general than the worked example. If the chapter feels easy while you are reading it but difficult when you close the page, you have not yet converted recognition into mastery.

The most effective study pattern is read, annotate, rebuild the worked example without looking, and then solve several short-to-long problems in one sitting so the idea becomes automatic.

Practice while you read

Practice Set: Double and triple integrals

Build region bounds cleanly and choose coordinates that match the geometry.

@@TOKEN_0@@ Evaluate from x=0 to 1 from y=0 to 2 of (x + y) dy dx.

- Hint: Integrate with respect to the inner variable first while treating x as constant.
- Step 1: Compute the inner integral 0^2 (x + y) dy.
- Step 2: Simplify the resulting function of x.
- Step 3: Integrate that expression from 0 to 1.
- Checkpoint: Integral value: 3

@@TOKEN_0@@ Convert the point (1, 3) to polar coordinates with r > 0.

- Hint: Find r from the distance to the origin and from the reference angle in the first quadrant.
- Step 1: Compute r = (x^2 + y^2).
- Step 2: Use tan() = y/x to identify the angle.
- Step 3: State the polar pair (r,).
- Checkpoint: (r,) = (2, /3)

Chapter homework

@@TOKEN_0@@ Region setup, coordinate changes, and theorem-driven vector calculus.

1. Reverse the order of integration for integral from 0 to 1 integral from y to 1 f(x,y) dx dy.
2. Evaluate double integral over the rectangle [0,2] x [0,1] of (x + 2y) dA.
3. Use cylindrical coordinates to describe the solid inside x^2 + y^2 = 9 and between z = 0 and z = 4.
4. Find a potential function for F = <2x, 2y, 2z>.

Answers for these homework problems appear in the back-of-book answer key.

Chapter summary and study notes

- Describe the region before writing the bounds.
- Use Jacobian factors correctly after changing coordinates.
- Check whether symmetry can simplify an integral before calculating.

Study tips

- Draw the region and label bounds before writing iterated integrals.
- Switch the integration order when the current order creates unnecessary pain.
- When changing coordinates, explain what the new small area or volume element represents.

Common traps

- Copying bounds from the sketch without checking which variable is outer and which is inner.
- Changing coordinates but forgetting the Jacobian factor.
- Treating the region as rectangular when it is not.

Family-level errors to watch for

- Starting algebra before identifying the governing definition or theorem.
- Dropping notation, units, or sign conventions in the middle of a calculation.
- Treating a symbolic answer as finished without interpreting what it means.

Chapter 4

Chapter 4 Line integrals, flux, and the major theorems

Chapter purpose

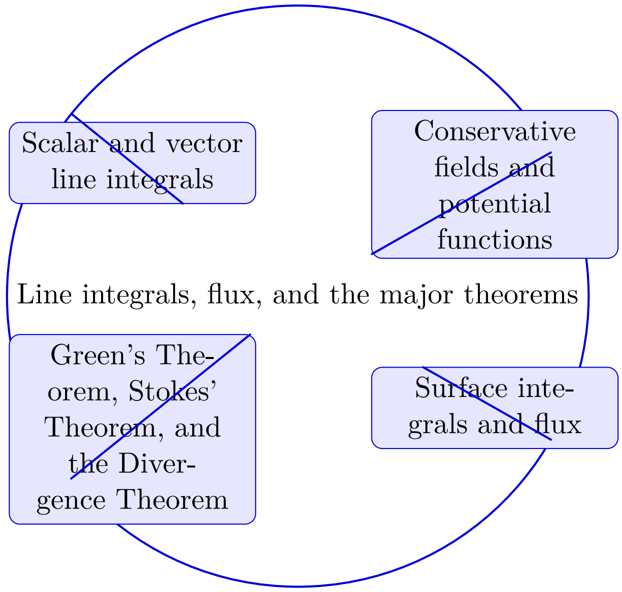
The course ends by tying local derivatives to global circulation and flux. Students compute line integrals, conservative fields, surface integrals, and use Green's, Stokes', and the Divergence Theorem. The aim is not theorem worship; it is learning when a theorem changes an ugly calculation into a clean geometric one.

This chapter sits at the end of Calculus III. It develops Scalar and vector line integrals, Conservative fields and potential functions, Surface integrals and flux, and Green's Theorem, Stokes' Theorem, and the Divergence Theorem so that the student can move from explanation to execution without losing the thread of the course.

The central habit in this chapter is to move across words, graphs, formulas, and worked algebra without losing meaning. A correct answer is not enough on its own; the student should be able to explain why the setup is valid and how the result fits the larger mathematical structure of the course.

Core ideas

- Scalar and vector line integrals
- Conservative fields and potential functions
- Surface integrals and flux
- Green's Theorem, Stokes' Theorem, and the Divergence Theorem



How to think through this chapter

Problem solving in this family starts with naming the structure of the task. Students should ask which theorem, definition, or representation controls the problem before choosing a computational path. Once the structure is clear, algebraic execution should be clean, annotated, and checked against the expected behavior of the function or model.

When working this chapter, keep the following question active: @@TOKEN_0@@ A good student answer should connect setup, assumptions, and conclusion instead of only chasing a final number or sentence.

Vector calculus is where multivariable calculus becomes a language for fields, flow, circulation, and conservation. The operators look compact, but each one is answering a very physical question about how a field behaves in space.

Divergence and curl are geometric diagnostics

Divergence asks whether a field behaves locally like a source or sink. Curl asks whether the field tends to rotate around a point. These are not abstract curiosities. They are compact tests for how a field moves matter, energy, or influence through a region.

Students understand these much better when they imagine tiny particles or paddle wheels responding to the field. The computation then supports the picture instead of replacing it.

Line and surface integrals measure interaction

A line integral asks what the field does along a path. A surface integral asks what passes through a surface. These are interaction questions, not simply area or length questions with extra notation.

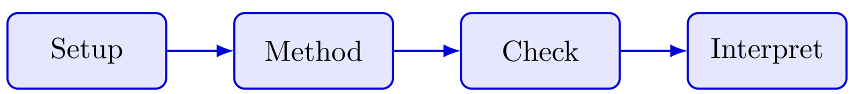
Once students see that, the later theorems of Green, Stokes, and Gauss become more believable. Those theorems are translation rules between local behavior and boundary behavior.

Big theorems are compression tools

The major integral theorems are powerful because they compress a complicated region computation into a cleaner boundary computation, or the reverse. They are not magic shortcuts detached from meaning. They rely on deep consistency between local change and global accumulation.

A practical study trick is to ask what shape is easier: the region or its boundary. That instinct often points directly to the right theorem.

Worked example



@@TOKEN_0@@ Determine whether F = <2xy, x^2 + 3y^2> is conservative in the plane.

1. Compute partial derivatives: dP/dy = 2x and dQ/dx = 2x.
2. The mixed partials agree on all of R^2, which is simply connected.
3. Therefore the field is conservative.
4. A potential function can be built by integrating P with respect to x.

Read this example twice: once for the flow of ideas and once for the technical structure of the solution.

Worked-through guided example

@@TOKEN_0@@ Find the divergence of F(x, y, z) = <x^2, yz, 3z>.

1. Differentiate x^2 with respect to x.
2. Differentiate yz with respect to y and 3z with respect to z.
3. Add the three results.

The divergence is 2x + z + 3 because /x(x^2) = 2x, /y(yz) = z, and /z(3z) = 3.

Instructor commentary

Students should annotate this chapter for structure, not just facts. Mark where the argument changes direction, where the method requires a hidden assumption, and where the conclusion becomes more general than the worked example. If the chapter feels easy while you are reading it but difficult when you close the page, you have not yet converted recognition into mastery.

The most effective study pattern is read, annotate, rebuild the worked example without looking, and then solve several short-to-long problems in one sitting so the idea becomes automatic.

Practice while you read

Practice Set: Divergence, curl, and field interpretation

Translate between symbolic operators and geometric field behavior.

@@TOKEN_0@@ Find the divergence of $F(x, y, z) = \langle x\hat{}2, yz, 3z \rangle$.

- Hint: Divergence is the sum of the matching partial derivatives of the vector field components.
- Step 1: Differentiate x^2 with respect to x.
- Step 2: Differentiate yz with respect to y and 3z with respect to z.
- Step 3: Add the three results.
- Checkpoint: div F = 2x + z + 3

@@TOKEN_0@@ Find the curl of $F(x, y, z) = \langle y, z, x \rangle$.

- Hint: Use the determinant-style formula for curl and compute each component carefully.
- Step 1: Write the i, j, and k component formulas for curl F.
- Step 2: Compute the needed partial derivatives of y, z, and x.
- Step 3: Assemble the final vector.
- Checkpoint: curl $F = \langle -1, -1, -1 \rangle$

Chapter homework

@@TOKEN_0@@ Region setup, coordinate changes, and theorem-driven vector calculus.

1. Reverse the order of integration for integral from 0 to 1 integral from y to 1 f(x,y) dx dy.
2. Evaluate double integral over the rectangle [0,2] x [0,1] of (x + 2y) dA.

3. Use cylindrical coordinates to describe the solid inside $x^2 + y^2 = 9$ and between $z = 0$ and $z = 4$.
4. Find a potential function for $F = <2x, 2y, 2z>$.

Answers for these homework problems appear in the back-of-book answer key.

Chapter summary and study notes

- Parameterize curves and surfaces with the correct orientation.
- Test for conservativeness before brute-force integration.
- Use theorems to simplify when symmetry or boundary structure makes them better than direct work.

Study tips

- Visualize divergence with sources and sinks, and curl with local spin.
- Before choosing a theorem, compare the complexity of the interior to the complexity of the boundary.
- Parameterize paths and surfaces with geometry in mind, not by guesswork alone.

Common traps

- Computing divergence or curl correctly but never interpreting what the result says physically.
- Using the wrong orientation on a curve or surface.
- Applying a major theorem without checking its hypotheses and the geometry of the boundary.

Family-level errors to watch for

- Starting algebra before identifying the governing definition or theorem.
- Dropping notation, units, or sign conventions in the middle of a calculation.
- Treating a symbolic answer as finished without interpreting what it means.

Chapter 5

Quiz review and official exam preparation

Homework structure

- Homework Set 1: 3D geometry and partial derivatives: 4 graded problems attached to chapter 1.
- Homework Set 2: Multiple integrals and vector fields: 4 graded problems attached to chapter 2.

Quiz structure

- Quiz 1: Vectors and multivariable derivatives: 4 questions, timed, and single-attempt in the live course. Quiz 1 should be taken only after you can solve the chapter homework without outside prompts.
- Quiz 2: Multiple integrals and vector calculus: 4 questions, timed, and single-attempt in the live course. Quiz 2 should be taken only after you can solve the chapter homework without outside prompts.

Official mastery exam

- Calculus III cumulative mastery exam: 5 major questions, High rigor, first official attempt locks the course grade.

Calculus III cumulative mastery exam preparation checklist

- Draw 3D objects before writing equations or bounds.
- Review both geometric meaning and computation for gradients and directional derivatives.

- Practice switching coordinate systems only when the geometry makes the change worthwhile.
- Know the hypotheses of the major theorems, not only their formulas.

How to use this book before assessment

- Read the relevant chapter and rebuild both worked examples without looking.
- Solve the guided practice in the chapter before attempting the graded homework.
- Check your chapter-homework answers only after you complete a full written attempt.
- Review the quiz answer key after each chapter block and classify your errors by concept, setup, algebra, or interpretation.
- Before the official exam, revisit the chapter purposes, homework corrections, and answer-key notes rather than rereading formulas only.

Chapter 6

Course vocabulary index

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Chapter 7

Back-of-book answers and solution outlines

Guided practice answer key

Chapter 1: Vectors and geometry in space

@@TOKEN_0@@

1. Find the dot product of a = <1, 2, -1> and b = <3, 0, 4>.

- Checkpoint answer: a ů b = -1 - Solution note: The dot product is 3 + 0 - 4 = -1, indicating slight opposite-direction alignment overall.

1. Find the projection of v = <4, 2> onto u = <1, 1>.

- Checkpoint answer: Projection = <3, 3> - Solution note: vůu = 6 and uůu = 2, so the factor is 3. The projection is 3<1, 1> = <3, 3>.

Chapter 2: Partial derivatives, gradients, and local linear models

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1. Find fx and fy for f(x, y) = x^2y + 3xy^2.

- Checkpoint answer: fx = 2xy + 3y^2, fy = x^2 + 6xy - Solution note: Treating y as constant gives fx = 2xy + 3y^2. Treating x as constant gives fy = x^2 + 6xy.

1. Find the gradient of f(x, y) = x^2 + y^2 at the point (1, -2).

- Checkpoint answer: f(1, -2) = <2, -4> - Solution note: The gradient is f = <2x, 2y>. At (1, -2) this becomes <2, -4>.

Chapter 3: Double integrals, triple integrals, and coordinate changes

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1. Evaluate from x=0 to 1 from y=0 to 2 of (x + y) dy dx.

- Checkpoint answer: Integral value: 3 - Solution note: The inner integral is 2x + 2. Integrating from 0 to 1 gives 0^1 (2x + 2) dx = [x^2 + 2x]0^1 = 3.

1. Convert the point (1, 3) to polar coordinates with r > 0.

- Checkpoint answer: (r,) = (2, /3) - Solution note: r = (1 + 3) = 2 and tan() = 3, so = /3 in the first quadrant.

Chapter 4: Line integrals, flux, and the major theorems

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1. Find the divergence of F(x, y, z) = <x^2, yz, 3z>.

- Checkpoint answer: div F = 2x + z + 3 - Solution note: The divergence is 2x + z + 3 because /x(x^2) = 2x, /y(yz) = z, and /z(3z) = 3.

1. Find the curl of F(x, y, z) = <y, z, x>.

- Checkpoint answer: curl F = <-1, -1, -1> - Solution note: curl F = <x/y - z/z, y/z - x/x, z/x - y/y> = <0 - 1, 0 - 1, 0 - 1> = <-1, -1, -1>.

Homework answer key

Homework Set 1: 3D geometry and partial derivatives

1. Find the angle between vectors <1,2,-1> and <2,0,3>.

- Answer / solution summary: Compute the dot product and divide by the product of magnitudes, then apply arccos.

1. Find the line of intersection between the planes x + y + z = 3 and x - y + z = 1.

- Answer / solution summary: Subtract to get 2y = 2, so y = 1. Then x + z = 2, giving the line <2 - t, 1, t>.

1. For f(x,y) = x e^(xy), find the directional derivative at (1,0) in the direction <3,4>.

- Answer / solution summary: Compute the gradient, evaluate at (1,0), and dot with <3/5,4/5>.

1. Find the tangent plane to z = x^2 + xy + y^2 at (1,1,3).

- Answer / solution summary: fx = 2x + y and fy = x + 2y, both equal 3 at (1,1). The plane is z - 3 = 3(x - 1) + 3(y - 1).

Homework Set 2: Multiple integrals and vector fields

1. Reverse the order of integration for integral from 0 to 1 integral from y to 1 f(x,y) dx dy.

- Answer / solution summary: The reversed form is integral from 0 to 1 integral from 0 to x f(x,y) dy dx.

1. Evaluate double integral over the rectangle [0,2] x [0,1] of (x + 2y) dA.

- Answer / solution summary: The value is 4.

1. Use cylindrical coordinates to describe the solid inside x^2 + y^2 = 9 and between z = 0 and z = 4.

- Answer / solution summary: 0 <= r <= 3, 0 <= theta <= 2pi, 0 <= z <= 4.

1. Find a potential function for F = <2x, 2y, 2z>.

- Answer / solution summary: A potential is phi = x^2 + y^2 + z^2 + C.

Quiz answer key

Quiz 1: Vectors and multivariable derivatives

1. Two vectors are orthogonal when their:

- Answer key: Dot product is zero. A zero dot product encodes a right angle between nonzero vectors.

1. If f(x,y) = x^2 + y^2, what is grad f at (1,2)? Enter as <a,b>.

- Answer key: Accepted answer(s): <2,4>, <2, 4>. grad f = <2x, 2y>, so at (1,2) it is <2,4>.

1. The tangent plane is the best local:

- Answer key: Linear model. The tangent plane is the first-order linear approximation.

1. Lagrange multipliers are used when:

- Answer key: An extremum is sought under a constraint. The method connects the gradient of the objective to the gradient of the constraint.

Quiz 2: Multiple integrals and vector calculus

1. When switching from Cartesian to polar coordinates, dA becomes:

- Answer key: r dr dtheta. The Jacobian contributes the factor r.

1. What is the value of line integral of grad phi over any path from A to B?

- Answer key: Accepted answer(s): phi(B)-phi(A), phi(b)-phi(a), the change in the potential. For conservative fields, the line integral depends only on endpoints.

1. Divergence measures the local tendency of a field to:

- Answer key: Spread out or compress. Positive divergence suggests local outflow; negative divergence suggests local compression.

1. Green's Theorem converts a line integral around a closed plane curve into a:

- Answer key: Double integral over the enclosed region. It exchanges circulation around the boundary with area accumulation inside the region.

Mastery exam solution outlines

Calculus III cumulative mastery exam

1. Find the shortest distance from the point (2,-1,4) to the plane 2x - y + 2z = 7.

- What to show: A point-to-plane distance setup or projection argument; A simplified exact answer - Solution outline: Use |ax0 + by0 + cz0 + d| / sqrt(a^2 + b^2 + c^2). Rewrite the plane as 2x - y + 2z - 7 = 0 and evaluate the formula.

1. Use Lagrange multipliers to maximize f(x,y) = xy subject to x^2 + y^2 = 8.

- What to show: The gradient equations and constraint; All candidate points and the maximizing value - Solution outline: Set <y, x> = lambda <2x, 2y>. Solve together with x^2 + y^2 = 8 to get candidates at (plus/minus 2, plus/minus 2). The maximum value is 4 at (2,2) and (-2,-2).

1. Evaluate double integral over the region bounded by y = x^2 and y = 2x of x dA.

- What to show: Intersection points and region bounds; An iterated integral in a justified order - Solution outline: Solve x^2 = 2x to get x = 0 and x = 2. Use integral from 0 to 2 integral from x^2 to 2x of x dy dx.

1. Compute the flux of F = <x,y,z> outward through the sphere x^2 + y^2 + z^2 = 9 using the Divergence Theorem.

- What to show: The divergence and the enclosed volume; A clear theorem-based conclusion - Solution outline: div F = 3. The sphere volume is 36pi. Flux is 3 times 36pi = 108pi.

1. Evaluate the circulation of F = <-y, x> around the unit circle oriented counterclockwise, and explain why a theorem is faster than direct parameterization here.

- What to show: A Green's Theorem setup; A conceptual explanation of why it simplifies the work - Solution outline: With P = -y and Q = x, dQ/dx - dP/dy = 1 - (-1) = 2. Green's Theorem gives integral over the unit disk of 2 dA = 2pi.

Reference note

For the full bibliography behind this textbook, use @@TOKEN_0@@. The answer key in this book is Summit-authored and aligned to the live course runtime.