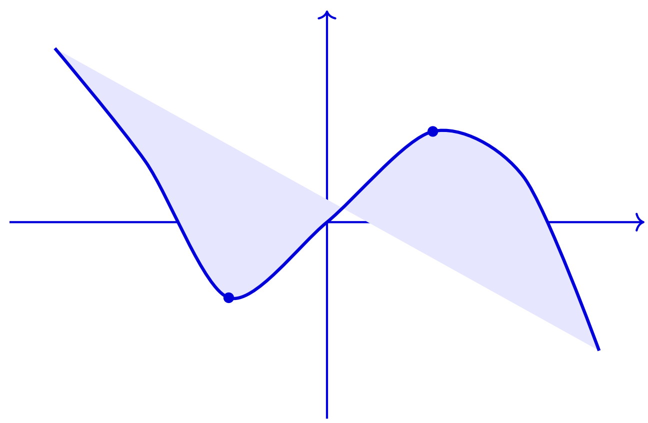
Summit MATH 152: Calculus II

Summit fully illustrated textbook edition



Original Summit-authored instructional text generated from the live course runtime, bibliography layer, and assessment structure.

March 22, 2026

@@TOKEN_0@@ Summit first edition draft @@TOKEN_1@@ college @@TOKEN_2@@ 4 @@TOKEN_3@@ 14 weeks @@TOKEN_4@@ 12.9 hours/week

Originality note

This textbook is a Summit-authored instructional text. It is informed by the course bibliography in @@TOKEN_0@@ and by open academic references used elsewhere in Summit, but it does not copy or restate any single commercial textbook.

How this textbook was built

This book was generated from the live Summit course runtime for Calculus II: the syllabus, lesson sequence, reading chapters, guided practice, homework sets, quizzes, mastery exam, and workload standard. The design goal is to give a student a usable, course-complete book while preserving original Summit wording and sequencing.

An original Summit second course in single-variable calculus focused on integration strategy, geometric and physical applications, sequences, series, power series, and alternate curve descriptions used in engineering.

Mathematics chapters should move from concept to representation to fluent execution. Students should always know what the symbols mean before they try to manipulate them.

This volume is structured as a teaching book rather than a bare note pack. Every chapter contains explanation, worked examples, guided practice, chapter homework, and a rear answer key so the student can study independently and still get disciplined feedback.

Course use guide

- Read one chapter at a time in sequence; each chapter is aligned to a live lesson block in the course workspace.
- Rebuild the worked examples before attempting the graded homework or quiz material.
- Keep a scratch notebook beside the text and write down assumptions, diagrams, and the points where you usually get stuck.
- Use the course tutor, guided practice, and homework only after you can explain the chapter in your own words.

Contents

Course map

- 4 live lesson chapters
- 2 graded homework checkpoints
- 2 timed quizzes
- 1 cumulative mastery exam
- 6 declared course outcomes

Prerequisite and readiness position

Course prerequisites: calculus-i. Readiness clearances: calc-i-credit.

Summit Calculus II assumes the student can already reason with limits, compute derivatives fluently, and use the Fundamental Theorem of Calculus correctly.

Semester workload standard

Summit models this course as @@TOKEN_0@@ across a 14-week term plus final assessment window. The expected distribution is:

- Contact-equivalent instruction: 56 hours
- Reading: 18 hours
- Practice and problem solving: 62 hours
- Homework: 24 hours
- Lab, design, and reporting: 0 hours
- Exam preparation: 20 hours

Expected volume:

- 170-210 problems spanning integration methods, applications of integration, sequences, series, and alternate curve models.
- 10 graded homework sets totaling 45-55 multistep problems with full written solutions.
- No standalone lab block; explanatory writeups are embedded inside homework, corrections, and exam review.

Reference basis

Primary synthesis anchors from the bibliography for this course (50 listed references total):

1. Calculus
2. Calculus
3. Thomas' Calculus
4. Calculus, Volume 1
5. Active Calculus
6. Calculus for Engineering Students
7. Applied Calculus for Scientists and Engineers
8. Introduction to Integral Calculus

Chapter 1

Chapter 1 Advanced integration techniques

Chapter purpose

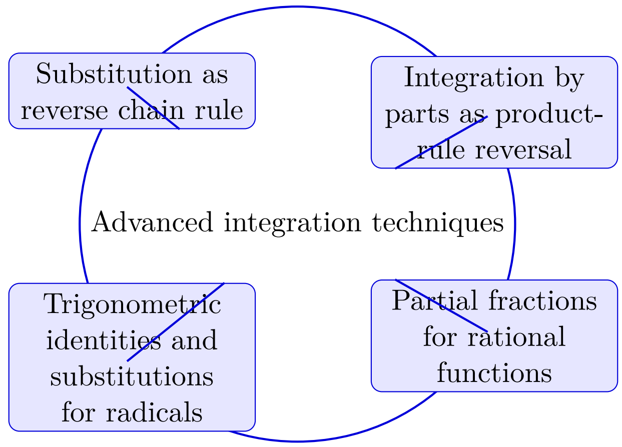
Calculus II opens by turning integration into strategy. Students compare substitution, integration by parts, trigonometric integrals, trig substitution, and partial fractions. The lesson emphasizes diagnosis first: before integrating, students should identify structure, likely transformations, and the algebra needed to finish the job cleanly.

This chapter sits at the opening of Calculus II. It develops Substitution as reverse chain rule, Integration by parts as product-rule reversal, Partial fractions for rational functions, and Trigonometric identities and substitutions for radicals so that the student can move from explanation to execution without losing the thread of the course.

The central habit in this chapter is to move across words, graphs, formulas, and worked algebra without losing meaning. A correct answer is not enough on its own; the student should be able to explain why the setup is valid and how the result fits the larger mathematical structure of the course.

Core ideas

- Substitution as reverse chain rule
- Integration by parts as product-rule reversal
- Partial fractions for rational functions
- Trigonometric identities and substitutions for radicals



How to think through this chapter

Problem solving in this family starts with naming the structure of the task. Students should ask which theorem, definition, or representation controls the problem before choosing a computational path. Once the structure is clear, algebraic execution should be clean, annotated, and checked against the expected behavior of the function or model.

When working this chapter, keep the following question active: @@TOKEN_0@@ A good student answer should connect setup, assumptions, and conclusion instead of only chasing a final number or sentence.

Calculus II begins with a practical reality: many integrals are not obvious. Integration techniques are not a list to memorize blindly. They are a pattern-recognition system for turning an ugly expression into one the Fundamental Theorem can use.

Technique choice is the real skill

Students often think success in this unit means remembering many formulas. The deeper skill is recognizing structure. Is there a composite function inviting substitution? Is there a product where one factor simplifies under repeated differentiation? Does a rational expression want partial fractions? Good technique choice saves far more time than algebra speed.

This is why worked examples should be read diagnostically. The key question is not only "what method did the solution use?" but "what clue in the integrand made that method a strong choice?"

Integration is less automatic than differentiation

Differentiation has a strong forward grammar. Given an expression, the next move is often obvious. Integration is more like reverse engineering. Several antiderivative paths may be possible, and some are much cleaner than others. That uncertainty is normal, not a sign the student is weak.

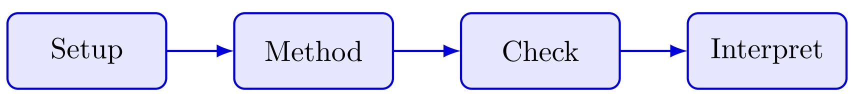
A good integrator becomes comfortable with short trial moves. Rewrite, factor, separate, or substitute just enough to see whether the structure improves. If it does not, back up early instead of plowing forward into a page of algebra.

Exactness matters because later applications depend on it

In many Calc II problems, a minor algebra slip destroys the antiderivative completely. There is less tolerance for loose symbolic work than there was in early derivative drills. That is frustrating at first, but it is also preparation for later engineering analysis where one sign error can reverse the physical interpretation of a model.

The right response is not panic. It is process discipline: carry constants carefully, check substitutions, and differentiate the answer whenever possible.

Worked example



@@TOKEN_0@@ Evaluate integral of x e^x dx.

1. Use integration by parts with u = x and dv = e^x dx.
2. Then du = dx and v = e^x.
3. Apply the formula: integral x e^x dx = x e^x - integral e^x dx.
4. The final answer is e^x(x - 1) + C.

Read this example twice: once for the flow of ideas and once for the technical structure of the solution.

Worked-through guided example

@@TOKEN_0@@ Evaluate integral of 2x cos(x^2) dx.

1. Let u = x^2 so du = 2x dx.
2. Rewrite the integral entirely in terms of u.
3. Integrate cos(u) and then substitute back.

With u = x^2, the integral becomes cos(u) du = sin(u) + C = sin(x^2) + C.

Instructor commentary

Students should annotate this chapter for structure, not just facts. Mark where the argument changes direction, where the method requires a hidden assumption, and where the conclusion becomes more general than the worked example. If the chapter feels easy while you are reading it but difficult when you close the page, you have not yet converted recognition into mastery.

The most effective study pattern is read, annotate, rebuild the worked example without looking, and then solve several short-to-long problems in one sitting so the idea becomes automatic.

Practice while you read

Practice Set: Integration technique choice

Practice recognizing whether a substitution, parts, or algebraic rewrite is the right first move.

@@TOKEN_0@@ Evaluate integral of 2x cos(x^2) dx.

- Hint: The inside of the cosine has a derivative already present outside.
- Step 1: Let u = x^2 so du = 2x dx.
- Step 2: Rewrite the integral entirely in terms of u.
- Step 3: Integrate cos(u) and then substitute back.
- Checkpoint: sin(x^2) + C

@@TOKEN_0@@ Evaluate integral of x e^x dx.

- Hint: Use integration by parts with the polynomial chosen as u so it becomes simpler when differentiated.
- Step 1: Set u = x and dv = e^x dx.
- Step 2: Then du = dx and v = e^x.
- Step 3: Apply u dv = uv - v du and simplify.
- Checkpoint: (x - 1)e^x + C

Chapter homework

@@TOKEN_0@@ Method selection and algebraic control inside nontrivial integrals.

1. Evaluate integral of x / (x^2 + 9) dx.
2. Evaluate integral of x^2 ln(x) dx.

3. Decompose and integrate 1 / [x(x + 1)].
4. Evaluate integral of dx / sqrt(16 - x^2).

Answers for these homework problems appear in the back-of-book answer key.

Chapter summary and study notes

- Choose a technique because of structure, not because it was used last class.
- Rewrite rational functions to proper form before partial fractions.
- Keep track of back-substitution carefully in trig-substitution problems.

Study tips

- Ask first whether a u-substitution would remove the hardest part of the integrand.
- For products, try integration by parts only after deciding which factor becomes simpler when differentiated.
- Differentiate the final answer whenever the algebra is messy enough to hide a sign error.

Common traps

- Choosing a technique because it appeared in the previous problem rather than because the structure matches.
- Forgetting to rewrite the entire integrand in terms of the substitution variable.
- Losing constants or signs in repeated integration by parts.

Family-level errors to watch for

- Starting algebra before identifying the governing definition or theorem.
- Dropping notation, units, or sign conventions in the middle of a calculation.
- Treating a symbolic answer as finished without interpreting what it means.

Chapter 2

Chapter 2 Applications of integration

Chapter purpose

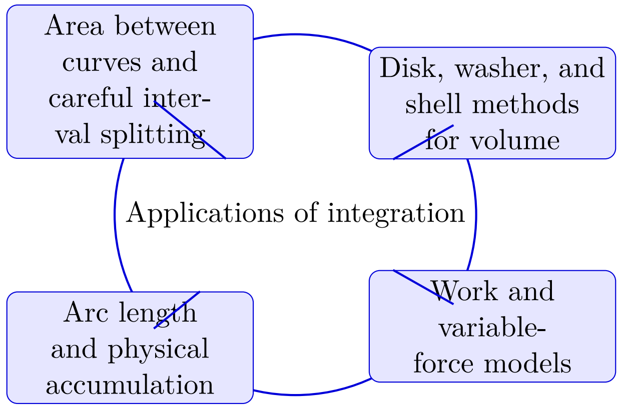
This lesson turns antiderivatives into modeling tools. Students compute planar area between curves, solids of revolution, work, hydrostatic force, center of mass, and arc length. The calculations matter, but setup is the real differentiator: students must choose slices, draw regions, and justify bounds and radii before any integral appears.

This chapter sits in the middle of Calculus II. It develops Area between curves and careful interval splitting, Disk, washer, and shell methods for volume, Work and variable-force models, and Arc length and physical accumulation so that the student can move from explanation to execution without losing the thread of the course.

The central habit in this chapter is to move across words, graphs, formulas, and worked algebra without losing meaning. A correct answer is not enough on its own; the student should be able to explain why the setup is valid and how the result fits the larger mathematical structure of the course.

Core ideas

- Area between curves and careful interval splitting
- Disk, washer, and shell methods for volume
- Work and variable-force models
- Arc length and physical accumulation



How to think through this chapter

Problem solving in this family starts with naming the structure of the task. Students should ask which theorem, definition, or representation controls the problem before choosing a computational path. Once the structure is clear, algebraic execution should be clean, annotated, and checked against the expected behavior of the function or model.

When working this chapter, keep the following question active: @@TOKEN_0@@ A good student answer should connect setup, assumptions, and conclusion instead of only chasing a final number or sentence.

Once integration techniques are available, the course turns them toward geometry, motion, and accumulation models that derivatives alone cannot finish. The central question is no longer "can you integrate this?" but "what quantity does the integral represent?"

Applications depend on choosing the right tiny piece

Area, volume, work, and arc length all come from the same architecture: slice the object into manageable pieces, express one piece mathematically, then add them all through integration. The slice is everything. A bad slice creates a bad integral, even if the later algebra is flawless.

This is why many strong solutions begin with a sketch and a sentence, not with an equation. The student first decides what the differential element stands for, then builds the integral around it.

Geometry and calculus are cooperating

Disk, washer, and shell methods can feel like competing formulas. They are better understood as geometric viewpoints. The calculus is the same: add many small contributions. The geometry decides what each contribution looks like.

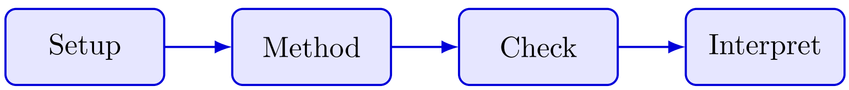
Students who keep the geometry in front of them remember formulas less by force and more by logic. They can even rebuild a forgotten formula during a test if they still understand the slice.

Units are a hidden guide

Applications of integration are full of unit checks. If radius has units of length and thickness has units of length, a disk contribution should have units of volume. If density is mass per length and dx is length, the product should look like mass. These checks are not optional polish. They catch setup errors before integration begins.

Students who work with units during setup usually make fewer conceptual mistakes and trust their answers for better reasons.

Worked example



@@TOKEN_0@@ Find the volume generated by revolving the region under y = x^2 from 0 to 1 about the x-axis.

1. Using disks, the radius is x^2.
2. Volume is pi integral from 0 to 1 of (x^2)^2 dx = pi integral from 0 to 1 of x^4 dx.
3. Integrate to get pi[x^5/5] from 0 to 1.
4. The volume is pi/5.

Read this example twice: once for the flow of ideas and once for the technical structure of the solution.

Worked-through guided example

@@TOKEN_0@@ Find the volume of the solid formed by revolving y = x^2 on 0 x 2 about the x-axis.

1. Use the disk formula V = [radius]^2 dx.
2. Here the radius is y = x^2, so the integrand becomes x^4.
3. Integrate from 0 to 2 and simplify.

V = 0^2 x^4 dx = [x^5/5]0^2 = 32/5.

Instructor commentary

Students should annotate this chapter for structure, not just facts. Mark where the argument changes direction, where the method requires a hidden assumption, and where the conclusion becomes more general than the worked example. If the chapter feels easy while you are reading it but difficult when you close the page, you have not yet converted recognition into mastery.

The most effective study pattern is read, annotate, rebuild the worked example without looking, and then solve several short-to-long problems in one sitting so the idea becomes automatic.

Practice while you read

Practice Set: Integral applications

Build geometric and physical setups carefully before integrating.

@@TOKEN_0@@ Find the volume of the solid formed by revolving y = x^2 on 0 x 2 about the x-axis.

- Hint: Revolving around the x-axis with slices perpendicular to the axis suggests the disk method.
- Step 1: Use the disk formula V = [radius]^2 dx.
- Step 2: Here the radius is y = x^2, so the integrand becomes x^4.
- Step 3: Integrate from 0 to 2 and simplify.
- Checkpoint: Volume = 32 / 5

@@TOKEN_0@@ A force F(x) = 4x + 1 moves an object from x = 0 to x = 3. Compute the work.

- Hint: Work is the definite integral of force over displacement when force is given as a function of position.
- Step 1: Write the work integral W = 0^3 (4x + 1) dx.
- Step 2: Find an antiderivative.
- Step 3: Evaluate at 3 and subtract the value at 0.
- Checkpoint: Work = 21

Chapter homework

@@TOKEN_0@@ Method selection and algebraic control inside nontrivial integrals.

1. Evaluate integral of x / (x^2 + 9) dx.

2. Evaluate integral of x^2 ln(x) dx.
3. Decompose and integrate 1 / [x(x + 1)].
4. Evaluate integral of dx / sqrt(16 - x^2).

Answers for these homework problems appear in the back-of-book answer key.

Chapter summary and study notes

- Draw the region before choosing a volume method.
- Use horizontal shells only when they simplify setup.
- Distinguish between geometric formulas and accumulation formulas.

Study tips

- Decide what one representative slice means before writing the integral.
- Sketch axis, bounds, and slice direction for every solids problem.
- Use units as an early warning system.

Common traps

- Mixing shell and washer logic in the same setup.
- Using top minus bottom or outer minus inner with the wrong orientation.
- Treating formulas as separate memorization tasks instead of as slice-based models.

Family-level errors to watch for

- Starting algebra before identifying the governing definition or theorem.
- Dropping notation, units, or sign conventions in the middle of a calculation.
- Treating a symbolic answer as finished without interpreting what it means.

Chapter 3

Chapter 3 Sequences, series, and power series

Chapter purpose

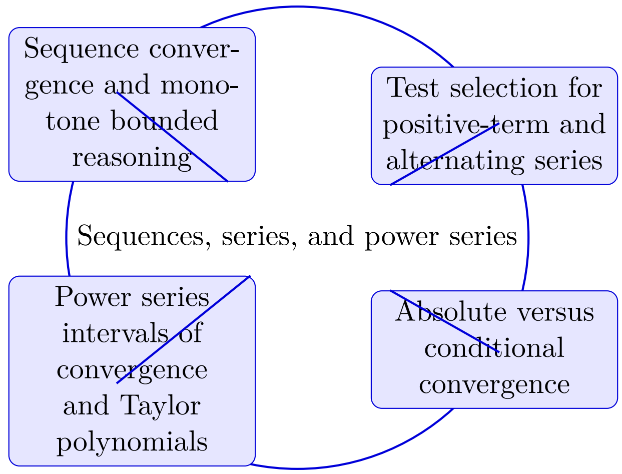
The course then shifts from finite accumulation to infinite processes. Students analyze limits of sequences, classify infinite series, and choose among divergence, integral, comparison, limit comparison, ratio, root, alternating, and absolute convergence tests. Power series and Taylor expansions extend the unit into approximation and local representation.

This chapter sits in the middle of Calculus II. It develops Sequence convergence and monotone bounded reasoning, Test selection for positive-term and alternating series, Absolute versus conditional convergence, and Power series intervals of convergence and Taylor polynomials so that the student can move from explanation to execution without losing the thread of the course.

The central habit in this chapter is to move across words, graphs, formulas, and worked algebra without losing meaning. A correct answer is not enough on its own; the student should be able to explain why the setup is valid and how the result fits the larger mathematical structure of the course.

Core ideas

- Sequence convergence and monotone bounded reasoning
- Test selection for positive-term and alternating series
- Absolute versus conditional convergence
- Power series intervals of convergence and Taylor polynomials



How to think through this chapter

Problem solving in this family starts with naming the structure of the task. Students should ask which theorem, definition, or representation controls the problem before choosing a computational path. Once the structure is clear, algebraic execution should be clean, annotated, and checked against the expected behavior of the function or model.

When working this chapter, keep the following question active: @@TOKEN_0@@ A good student answer should connect setup, assumptions, and conclusion instead of only chasing a final number or sentence.

Infinite series teach students to take an impossible object seriously. A series asks whether infinitely many additions can still settle to a finite value, and if so, how confidently we can use that value.

Convergence is about behavior, not arithmetic endurance

Students often meet series by calculating partial sums and hoping a pattern appears. That is a useful beginning, but the real issue is structural. The terms of the series have to decay in a way that allows the accumulated total to stabilize. Convergence tests are different lenses for judging that decay.

This is why no single test owns the whole unit. Ratio, root, comparison, and integral tests each notice different kinds of behavior. Choosing among them is similar to choosing an integration technique: diagnose the structure first.

Approximation turns infinite ideas into usable tools

Power series and Taylor polynomials show why this chapter matters beyond theory. Engineers routinely replace difficult functions with polynomial approximations because polynomials are easier to evaluate, differentiate, integrate, and embed in computations.

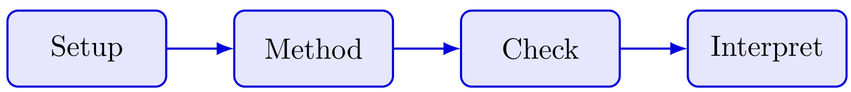
The important habit is to keep approximation error in view. A partial sum is not the full function. It is a controlled stand-in, and its usefulness depends on how the remainder behaves on the interval of interest.

Series reward qualitative judgment

A good series student develops an instinct for pace. Harmonic terms decay too slowly. Geometric terms are simple and decisive. Factorials dominate exponentials. These instincts make the formal tests feel natural instead of mysterious.

That judgment builds only if the student resists turning every series into a lookup exercise. It is worth spending time asking what family of behavior a new series resembles before choosing a test.

Worked example



@@TOKEN_0@@ Determine whether sum from n=1 to infinity of 1 / n^2 converges.

1. Recognize a p-series with p = 2.
2. A p-series converges when p > 1.
3. Therefore the series converges.
4. Because all terms are positive, this is absolute convergence as well.

Read this example twice: once for the flow of ideas and once for the technical structure of the solution.

Worked-through guided example

@@TOKEN_0@@ Determine whether from n=1 to of 1 / n^2 converges.

1. Identify the series as 1 / n^p with p = 2.
2. Recall that p-series converge when p > 1.
3. State the conclusion clearly.

Because this is a p-series with p = 2 > 1, the series converges.

Instructor commentary

Students should annotate this chapter for structure, not just facts. Mark where the argument changes direction, where the method requires a hidden assumption, and where the conclusion becomes more general than the worked example. If the chapter feels easy while you are reading it but difficult when you close the page, you have not yet converted recognition into mastery.

The most effective study pattern is read, annotate, rebuild the worked example without looking, and then solve several short-to-long problems in one sitting so the idea becomes automatic.

Practice while you read

Practice Set: Series and convergence

Recognize convergence behavior and connect tests to the structure of the series.

@@TOKEN_0@@ Determine whether from n=1 to of 1 / n^2 converges.

- Hint: This is a p-series, so compare the exponent to the p-series convergence rule.
- Step 1: Identify the series as 1 / n^p with p = 2.
- Step 2: Recall that p-series converge when p > 1.
- Step 3: State the conclusion clearly.
- Checkpoint: The series converges

@@TOKEN_0@@ Find the interval of convergence of from n=0 to of x^n.

- Hint: This is the geometric-series template with ratio r = x.
- Step 1: A geometric series converges when |r| < 1.
- Step 2: Here r = x, so impose |x| < 1.
- Step 3: Check the endpoints x = 1 and x = -1 separately.
- Checkpoint: Interval of convergence: (-1, 1)

Chapter homework

@@TOKEN_0@@ Model-building for applied integrals and careful convergence arguments.

1. Find the volume formed when the region between y = x and y = x^2 on [0,1] is revolved about the y-axis using shells.
2. Determine whether sum from n=2 to infinity of 1 / [n ln(n)] converges or diverges.

3. Find the interval of convergence for sum from n=0 to infinity of x^n / 3^n.
4. Compute the area inside r = 4 cos(theta) and outside r = 2.

Answers for these homework problems appear in the back-of-book answer key.

Chapter summary and study notes

- Name the reason a convergence test applies before computing with it.
- Check endpoint behavior after finding a radius of convergence.
- Interpret Taylor approximations as local models with truncation error.

Study tips

- Start by asking what the general term resembles asymptotically.
- If a test is inconclusive, treat that as information about the structure rather than as failure.
- For Taylor work, compare the approximation interval to the radius of convergence before trusting a polynomial.

Common traps

- Forgetting that the terms must go to zero before a series can even hope to converge.
- Applying a convergence test mechanically without checking its hypotheses.
- Assuming a polynomial approximation is accurate far outside the interval where it was built.

Family-level errors to watch for

- Starting algebra before identifying the governing definition or theorem.
- Dropping notation, units, or sign conventions in the middle of a calculation.
- Treating a symbolic answer as finished without interpreting what it means.

Chapter 4

Chapter 4 Parametric, vector, and polar calculus

Chapter purpose

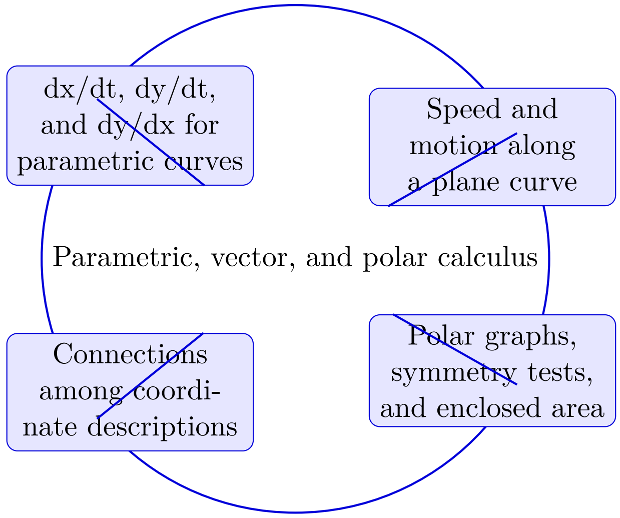
The final lesson packages motion and geometry in alternate coordinate systems. Students differentiate and integrate parametric curves, compute slope and concavity, and calculate polar area and tangent information. The main goal is flexibility: students should move between rectangular, parametric, and polar descriptions without losing the underlying geometry.

This chapter sits at the end of Calculus II. It develops dx/dt, dy/dt, and dy/dx for parametric curves, Speed and motion along a plane curve, Polar graphs, symmetry tests, and enclosed area, and Connections among coordinate descriptions so that the student can move from explanation to execution without losing the thread of the course.

The central habit in this chapter is to move across words, graphs, formulas, and worked algebra without losing meaning. A correct answer is not enough on its own; the student should be able to explain why the setup is valid and how the result fits the larger mathematical structure of the course.

Core ideas

- dx/dt, dy/dt, and dy/dx for parametric curves
- Speed and motion along a plane curve
- Polar graphs, symmetry tests, and enclosed area
- Connections among coordinate descriptions



How to think through this chapter

Problem solving in this family starts with naming the structure of the task. Students should ask which theorem, definition, or representation controls the problem before choosing a computational path. Once the structure is clear, algebraic execution should be clean, annotated, and checked against the expected behavior of the function or model.

When working this chapter, keep the following question active: @@TOKEN_0@@ A good student answer should connect setup, assumptions, and conclusion instead of only chasing a final number or sentence.

Parametric and polar forms teach students that not every curve wants to be written as y equals f of x. Sometimes geometry becomes clearer when position is described by time, angle, or another driving parameter.

A parameter tells a story

In a parametric curve, the variable is not just a placeholder. It often represents motion, phase, or progression through a process. That means the same point on the plane can arrive at a different speed or orientation depending on how the parameter behaves.

Students should read parametric equations dynamically. Ask where the curve starts, where it moves next, and what happens when the parameter increases. That perspective makes derivatives and arc-length questions more meaningful.

Polar coordinates trade rectangular convenience for symmetry

Polar equations are especially useful when a curve is built around rotation or radial distance. Circles, roses, spirals, and cardioids often look awkward in rectangular form but natural in polar form. The coordinate system is doing conceptual work, not just changing notation.

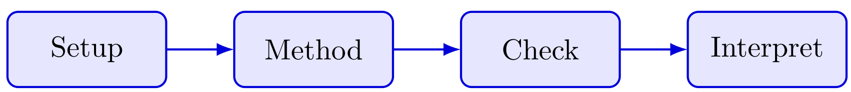
A strong trick here is to test symmetry early. Replacing theta with negative theta, pi minus theta, or theta plus pi often reveals the entire graph family before detailed plotting begins.

New coordinates still demand old discipline

Students sometimes relax too much when a new coordinate system appears. But the same core calculus habits still matter: identify what variable controls change, interpret derivatives carefully, and check whether the graph or region description matches the algebra.

The better mindset is not "this is a new topic with new tricks only." It is "this is the same calculus, now expressed in coordinates that fit the geometry better."

Worked example



@@TOKEN_0@@ Find the area enclosed by one loop of r = 2 sin(theta).

1. One loop occurs for theta from 0 to pi.
2. Polar area is (1/2) integral from 0 to pi of r^2 dtheta.
3. Substitute r = 2 sin(theta) to get 2 integral from 0 to pi of sin^2(theta) dtheta.
4. Using the half-angle identity, the area is pi.

Read this example twice: once for the flow of ideas and once for the technical structure of the solution.

Worked-through guided example

@@TOKEN_0@@ For x = t^2 and y = t^3, find dy/dx in terms of t.

1. Differentiate x and y with respect to t.
2. Form the ratio (dy/dt) / (dx/dt).
3. Simplify the resulting expression.

dx/dt = 2t and dy/dt = 3t^2, so dy/dx = (3t^2)/(2t) = (3/2)t when t 0.

Instructor commentary

Students should annotate this chapter for structure, not just facts. Mark where the argument changes direction, where the method requires a hidden assumption, and where the conclusion becomes more general than the worked example. If the chapter feels easy while you are reading it but difficult when you close the page, you have not yet converted recognition into mastery.

The most effective study pattern is read, annotate, rebuild the worked example without looking, and then solve several short-to-long problems in one sitting so the idea becomes automatic.

Practice while you read

Practice Set: Parametric and polar curves

Interpret alternate curve descriptions without defaulting back to rectangular form too early.

@@TOKEN_0@@ For x = t^2 and y = t^3, find dy/dx in terms of t.

- Hint: For parametric curves, use dy/dx = (dy/dt) / (dx/dt) when dx/dt is not zero.
- Step 1: Differentiate x and y with respect to t.
- Step 2: Form the ratio (dy/dt) / (dx/dt).
- Step 3: Simplify the resulting expression.
- Checkpoint: dy/dx = (3/2)t

@@TOKEN_0@@ Find the area enclosed by one loop of r = 2 sin(theta).

- Hint: Use the polar-area formula with the interval that traces the loop once.
- Step 1: Write A = (1/2) r^2 d.
- Step 2: Square r = 2 sin(theta) and choose the interval 0   .
- Step 3: Integrate 2 sin^2(theta) over that interval.
- Checkpoint: Area =

Chapter homework

@@TOKEN_0@@ Model-building for applied integrals and careful convergence arguments.

1. Find the volume formed when the region between y = x and y = x^2 on [0,1] is revolved about the y-axis using shells.
2. Determine whether sum from n=2 to infinity of 1 / [n ln(n)] converges or diverges.

3. Find the interval of convergence for sum from n=0 to infinity of x^n / 3^n.
4. Compute the area inside r = 4 cos(theta) and outside r = 2.

Answers for these homework problems appear in the back-of-book answer key.

Chapter summary and study notes

- Use dy/dx = (dy/dt) / (dx/dt) correctly.
- Test polar symmetry with purpose rather than guessing from shape.
- Select correct bounds when a curve traces itself more than once.

Study tips

- Build a small value table and indicate direction of motion for parametric curves.
- Check polar symmetry before plotting many points.
- Translate to rectangular form only when it actually simplifies the question.

Common traps

- Plotting a parametric curve without considering orientation.
- Assuming negative radius behaves like a standard rectangular negative sign.
- Forgetting that one geometric point can be represented by many polar coordinate pairs.

Family-level errors to watch for

- Starting algebra before identifying the governing definition or theorem.
- Dropping notation, units, or sign conventions in the middle of a calculation.
- Treating a symbolic answer as finished without interpreting what it means.

Chapter 5

Quiz review and official exam preparation

Homework structure

- Homework Set 1: Integration methods: 4 graded problems attached to chapter 1.
- Homework Set 2: Applications and infinite series: 4 graded problems attached to chapter 2.

Quiz structure

- Quiz 1: Integration strategy: 4 questions, timed, and single-attempt in the live course. Quiz 1 should be taken only after you can solve the chapter homework without outside prompts.
- Quiz 2: Series and polar curves: 4 questions, timed, and single-attempt in the live course. Quiz 2 should be taken only after you can solve the chapter homework without outside prompts.

Official mastery exam

- Calculus II cumulative mastery exam: 5 major questions, High rigor, first official attempt locks the course grade.

Calculus II cumulative mastery exam preparation checklist

- Practice choosing the method before doing any algebra.
- Review both shell and washer setups, especially around off-axis rotations.
- Memorize the logic of each convergence test and what information it actually gives.
- Work polar-area problems with attention to tracing intervals.

How to use this book before assessment

- Read the relevant chapter and rebuild both worked examples without looking.
- Solve the guided practice in the chapter before attempting the graded homework.
- Check your chapter-homework answers only after you complete a full written attempt.
- Review the quiz answer key after each chapter block and classify your errors by concept, setup, algebra, or interpretation.
- Before the official exam, revisit the chapter purposes, homework corrections, and answer-key notes rather than rereading formulas only.

Chapter 6

Course vocabulary index

- @@TOKEN_0@@: treat this as a working term in the course. You should be able to define it, recognize where it appears, and use it correctly in a solution or explanation.
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Chapter 7

Back-of-book answers and solution outlines

Guided practice answer key

Chapter 1: Advanced integration techniques

@@TOKEN_0@@

1. Evaluate integral of 2x cos(x^2) dx.

- Checkpoint answer: sin(x^2) + C - Solution note: With u = x^2, the integral becomes cos(u) du = sin(u) + C = sin(x^2) + C.

1. Evaluate integral of x e^x dx.

- Checkpoint answer: (x - 1)e^x + C - Solution note: Integration by parts gives xe^x - e^x dx = xe^x - e^x + C = (x - 1)e^x + C.

Chapter 2: Applications of integration

@@TOKEN_0@@

1. Find the volume of the solid formed by revolving y = x^2 on 0 x 2 about the x-axis.

- Checkpoint answer: Volume = 32 / 5 - Solution note: V = 0^2 x^4 dx = [x^5/5]0^2 = 32/5.

1. A force F(x) = 4x + 1 moves an object from x = 0 to x = 3. Compute the work.

- Checkpoint answer: Work = 21 - Solution note: W = [2x^2 + x]0^3 = (18 + 3) - 0 = 21.

Chapter 3: Sequences, series, and power series

@@TOKEN_0@@

1. Determine whether from n=1 to of 1 / n^2 converges.

- Checkpoint answer: The series converges - Solution note: Because this is a p-series with p = 2 > 1, the series converges.

1. Find the interval of convergence of from n=0 to of x^n.

- Checkpoint answer: Interval of convergence: (-1, 1) - Solution note: The geometric series converges for |x| < 1. At x = 1 the series is 1 + 1 + 1 + ... and diverges; at x = -1 it oscillates and diverges. So the interval is (-1, 1).

Chapter 4: Parametric, vector, and polar calculus

@@TOKEN_0@@

1. For x = t^2 and y = t^3, find dy/dx in terms of t.

- Checkpoint answer: dy/dx = (3/2)t - Solution note: dx/dt = 2t and dy/dt = 3t^2, so dy/dx = (3t^2)/(2t) = (3/2)t when t 0.

1. Find the area enclosed by one loop of r = 2 sin(theta).

- Checkpoint answer: Area = - Solution note: A = (1/2)0^ 4 sin^2 d = 20^ sin^2 d = 2(/2) = .

Homework answer key

Homework Set 1: Integration methods

1. Evaluate integral of x / (x^2 + 9) dx.

- Answer / solution summary: Let u = x^2 + 9, du = 2x dx. The antiderivative is (1/2) ln(x^2 + 9) + C.

1. Evaluate integral of x^2 ln(x) dx.

- Answer / solution summary: Take u = ln(x), dv = x^2 dx. The answer is (x^3 / 3)ln(x) - x^3 / 9 + C.

1. Decompose and integrate 1 / [x(x + 1)].

- Answer / solution summary: The decomposition is 1/x - 1/(x+1), so the antiderivative is ln|x| - ln|x+1| + C.

1. Evaluate integral of dx / sqrt(16 - x^2).

- Answer / solution summary: The antiderivative is arcsin(x/4) + C.

Homework Set 2: Applications and infinite series

1. Find the volume formed when the region between y = x and y = x^2 on [0,1] is revolved about the y-axis using shells.

- Answer / solution summary: Volume is 2pi integral from 0 to 1 of x(x - x^2) dx = pi/6.

1. Determine whether sum from n=2 to infinity of 1 / [n ln(n)] converges or diverges.

- Answer / solution summary: Integral of 1/(x ln x) is ln(ln x), which diverges, so the series diverges.

1. Find the interval of convergence for sum from n=0 to infinity of x^n / 3^n.

- Answer / solution summary: This is geometric with ratio x/3, so convergence requires |x| < 3. Endpoints diverge.

1. Compute the area inside r = 4 cos(theta) and outside r = 2.

- Answer / solution summary: Solve 4 cos(theta) = 2 to get theta = plus/minus pi/3. Use symmetry and the polar-area formula.

Quiz answer key

Quiz 1: Integration strategy

1. Which method is most natural for integral of ln(x) dx?

- Answer key: Integration by parts. Treat ln(x) as the part to differentiate and 1 as the part to integrate.

1. Evaluate integral from 0 to 1 of 3x^2 dx.

- Answer key: Accepted answer(s): 1, 1.0. An antiderivative is x^3, so the value is 1.

1. If a rational function has numerator degree greater than or equal to denominator degree, the first algebra step should be:

- Answer key: Perform polynomial long division. Partial fractions requires a proper rational function first.

1. Washer and shell methods produce the same physical volume when:

- Answer key: The same solid is being described correctly. They are alternative ways to compute the same solid when setup is correct.

Quiz 2: Series and polar curves

1. Which test is inconclusive for sum of 1/n?

- Answer key: Ratio test. The ratio test gives a limit of 1 for the harmonic series, so it says nothing.

1. A series that converges absolutely must also:

- Answer key: Converge. Absolute convergence is stronger than ordinary convergence.

1. What is the radius of convergence of sum from n=0 to infinity of x^n / 5^n?

- Answer key: Accepted answer(s): 5, 5.0. This is geometric with ratio x/5, so the radius is 5.

1. Polar area uses the factor 1/2 because:

- Answer key: Each sector is approximated by a triangle with area 1/2 r^2 dtheta. Polar area comes from summing tiny sector-like triangles.

Mastery exam solution outlines

Calculus II cumulative mastery exam

1. Evaluate integral of x^2 / (x^2 + 4) dx by rewriting the integrand into a more useful form before integrating.

- What to show: An algebraic decomposition before integration; A complete antiderivative - Solution outline: Rewrite as 1 - 4/(x^2 + 4). Integrate to get x - 2 arctan(x/2) + C.

1. Set up and evaluate the work required to pump all water from a cylindrical tank of radius 3 and height 10 if the tank is full and the outlet is 2 feet above the top.

- What to show: A differential slice with correct volume and lift distance; Units in the final result - Solution outline: Use a horizontal slice of thickness dy with volume 9pi dy. The lift distance for a slice at height y is 12 - y. Integrate weight density times slice volume times distance over the tank height.

1. Determine whether sum from n=1 to infinity of (3n + 1) / (n^2 + 4) converges or diverges and justify the test choice.

- What to show: Asymptotic comparison to a simpler benchmark; A correct conclusion - Solution outline: The terms behave like 3/n for large n. Use limit comparison with 1/n; the limit is 3, so the series diverges.

1. Find the Taylor polynomial of degree 4 for e^x centered at 0 and use it to approximate e^0.2.

- What to show: The polynomial itself; A clean numeric substitution - Solution outline: The degree-4 Maclaurin polynomial is 1 + x + x^2/2 + x^3/6 + x^4/24. Substitute x = 0.2 to get an approximation close to 1.2214.

1. Find the area common to the inside of r = 2 + 2 cos(theta) and the outside of r = 2.

- What to show: Intersection angles and a justified interval; Outer-minus-inner polar area setup - Solution outline: Solve 2 + 2 cos(theta) = 2 to get cos(theta) = 0, so theta = plus/minus pi/2. Integrate (1/2)[(2 + 2 cos(theta))^2 - 4] over the interval where the cardioid lies outside the circle.

Reference note

For the full bibliography behind this textbook, use @@TOKEN_0@@. The answer key in this book is Summit-authored and aligned to the live course runtime.