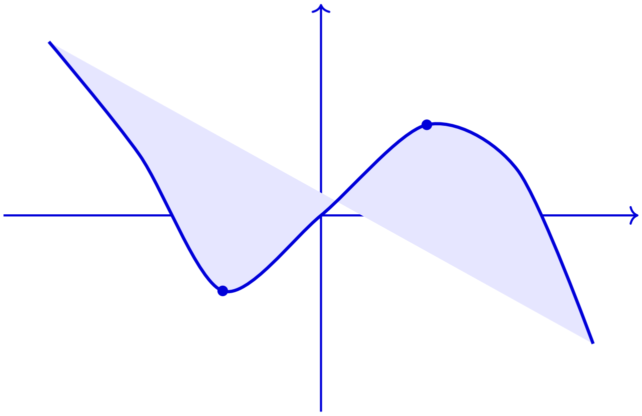
Summit MATH 151: Calculus I

Summit fully illustrated textbook edition



Original Summit-authored instructional text generated from the live course runtime, bibliography layer, and assessment structure.

March 22, 2026

@@TOKEN_0@@ Summit first edition draft @@TOKEN_1@@ college @@TOKEN_2@@ 4 @@TOKEN_3@@ 14 weeks @@TOKEN_4@@ 12.9 hours/week

Originality note

This textbook is a Summit-authored instructional text. It is informed by the course bibliography in @@TOKEN_0@@ and by open academic references used elsewhere in Summit, but it does not copy or restate any single commercial textbook.

How this textbook was built

This book was generated from the live Summit course runtime for Calculus I: the syllabus, lesson sequence, reading chapters, guided practice, homework sets, quizzes, mastery exam, and workload standard. The design goal is to give a student a usable, course-complete book while preserving original Summit wording and sequencing.

An original Summit course in single-variable calculus for engineering students. It develops functions, limits, continuity, derivatives, derivative applications, and a rigorous bridge into definite integrals and the Fundamental Theorem of Calculus.

Mathematics chapters should move from concept to representation to fluent execution. Students should always know what the symbols mean before they try to manipulate them.

This volume is structured as a teaching book rather than a bare note pack. Every chapter contains explanation, worked examples, guided practice, chapter homework, and a rear answer key so the student can study independently and still get disciplined feedback.

Course use guide

- Read one chapter at a time in sequence; each chapter is aligned to a live lesson block in the course workspace.
- Rebuild the worked examples before attempting the graded homework or quiz material.
- Keep a scratch notebook beside the text and write down assumptions, diagrams, and the points where you usually get stuck.
- Use the course tutor, guided practice, and homework only after you can explain the chapter in your own words.

Contents

Course map

- 4 live lesson chapters
- 2 graded homework checkpoints
- 2 timed quizzes
- 1 cumulative mastery exam
- 6 declared course outcomes

Prerequisite and readiness position

Readiness clearances: precalculus-ready.

Summit Calculus I begins the launch math sequence. Students do not need prior calculus credit, but they do need dependable algebra, function interpretation, geometry, and trigonometry before starting.

Semester workload standard

Summit models this course as @@TOKEN_0@@ across a 14-week term plus final assessment window. The expected distribution is:

- Contact-equivalent instruction: 56 hours
- Reading: 18 hours
- Practice and problem solving: 62 hours
- Homework: 24 hours
- Lab, design, and reporting: 0 hours
- Exam preparation: 20 hours

Expected volume:

- 170-210 short and multi-step calculus problems, including setup, symbolic work, and interpretation.
- 10 graded homework sets totaling 45-55 multistep problems with full written solutions.
- No standalone lab block; explanatory writeups are embedded inside homework, corrections, and exam review.

Reference basis

Primary synthesis anchors from the bibliography for this course (50 listed references total):

1. Calculus
2. Calculus
3. Thomas' Calculus
4. Calculus, Volume 1
5. Active Calculus
6. Calculus for Engineering Students
7. Applied Calculus for Scientists and Engineers
8. Introduction to Integral Calculus

Chapter 1

Chapter 1 Functions, limits, and continuity

Chapter purpose

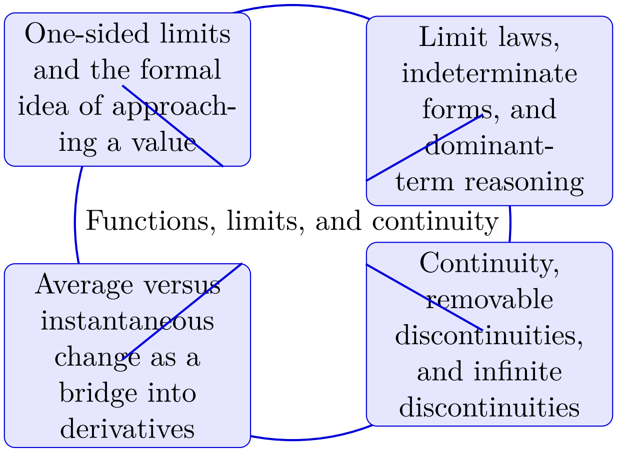
Students begin by treating a function as an object that can be represented numerically, graphically, verbally, and algebraically. The lesson pushes past plug-and-chug substitution into one-sided limits, continuity classes, asymptotic behavior, and careful limit laws. By the end, students should be able to explain why a limit exists, not just compute one.

This chapter sits at the opening of Calculus I. It develops One-sided limits and the formal idea of approaching a value, Limit laws, indeterminate forms, and dominant-term reasoning, Continuity, removable discontinuities, and infinite discontinuities, and Average versus instantaneous change as a bridge into derivatives so that the student can move from explanation to execution without losing the thread of the course.

The central habit in this chapter is to move across words, graphs, formulas, and worked algebra without losing meaning. A correct answer is not enough on its own; the student should be able to explain why the setup is valid and how the result fits the larger mathematical structure of the course.

Core ideas

- One-sided limits and the formal idea of approaching a value
- Limit laws, indeterminate forms, and dominant-term reasoning
- Continuity, removable discontinuities, and infinite discontinuities
- Average versus instantaneous change as a bridge into derivatives



How to think through this chapter

Problem solving in this family starts with naming the structure of the task. Students should ask which theorem, definition, or representation controls the problem before choosing a computational path. Once the structure is clear, algebraic execution should be clean, annotated, and checked against the expected behavior of the function or model.

When working this chapter, keep the following question active: @@TOKEN_0@@ A good student answer should connect setup, assumptions, and conclusion instead of only chasing a final number or sentence.

Limits matter because calculus studies motion and change at the instant where direct measurement breaks down. This chapter teaches students to stop asking only "what value do I plug in?" and start asking "what behavior is the function heading toward?"

Why limits arrive before derivatives

The derivative is supposed to measure an instantaneous rate of change. The problem is that "instantaneous" is not directly available. If two times are different, the rate is average. If the two times are identical, the denominator becomes zero. Limits are the tool that let calculus move toward the instant without falling into division by zero.

That is why the early weeks of Calculus I feel slower than students expect. The course is not avoiding applications. It is building the language that makes every later idea precise. Once students understand how nearby values behave, tangent slopes, velocity, and later integrals stop feeling like separate tricks.

How strong students think about a limit

A strong limit student looks at the form before doing algebra. If direct substitution gives a perfectly ordinary number, the problem is probably finished. If it gives zero over zero, infinity over infinity, or a graph with conflicting one-sided behavior, the student recognizes that the expression is sending a message: simplify me, compare growth, or inspect the left and right sides separately.

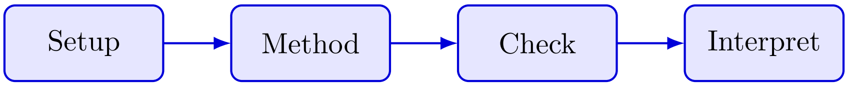
This is the first useful calculus habit: diagnose before computing. In algebra, students often start moving symbols immediately. In calculus, it is usually better to pause and classify the obstacle. That small pause prevents most wasted work.

What continuity is really saying

Continuity is not just "no holes in the graph." It is the statement that the output of the function matches the value the nearby behavior predicts. A function can be defined at a point and still fail to be continuous there if the nearby values head somewhere else. It can also have a removable hole, where the behavior is well behaved even though the function definition missed the target.

This viewpoint matters because later calculus depends on trust in local behavior. When a function is continuous, small input changes lead to controlled output changes. That is exactly the kind of stability that makes approximation, tangent-line reasoning, and accumulation possible.

Worked example



@@TOKEN_0@@ Evaluate lim x->2 (x^2 - 4) / (x - 2) and explain why substitution fails first.

1. Direct substitution gives 0 / 0, so the expression is indeterminate rather than undefined for all nearby x.
2. Factor the numerator: (x^2 - 4) = (x - 2)(x + 2).
3. Cancel the common factor for x not equal to 2, leaving x + 2.
4. Now the nearby behavior is controlled by x + 2, so the limit is 4.

Read this example twice: once for the flow of ideas and once for the technical structure of the solution.

Worked-through guided example

@@TOKEN_0@@ Evaluate lim x->3 (x^2 - 9) / (x - 3).

1. Factor the numerator as (x - 3)(x + 3).
2. Cancel the common factor for x not equal to 3 so the nearby expression becomes x + 3.

3. Now evaluate the remaining expression at x = 3.

The original form is indeterminate, but factoring removes the zero-over-zero obstacle. After cancellation the limit is lim x->3 (x + 3) = 6.

Instructor commentary

Students should annotate this chapter for structure, not just facts. Mark where the argument changes direction, where the method requires a hidden assumption, and where the conclusion becomes more general than the worked example. If the chapter feels easy while you are reading it but difficult when you close the page, you have not yet converted recognition into mastery.

The most effective study pattern is read, annotate, rebuild the worked example without looking, and then solve several short-to-long problems in one sitting so the idea becomes automatic.

Practice while you read

Practice Set: Limits and continuity

Train substitution judgment, algebraic cleanup, and continuity language before the first homework checkpoint.

@@TOKEN_0@@ Evaluate lim x->3 (x^2 - 9) / (x - 3).

- Hint: Direct substitution gives 0 / 0, so the expression wants a factorization before the limit is taken.
- Step 1: Factor the numerator as (x - 3)(x + 3).
- Step 2: Cancel the common factor for x not equal to 3 so the nearby expression becomes x + 3.
- Step 3: Now evaluate the remaining expression at x = 3.
- Checkpoint: Final answer: 6

@@TOKEN_0@@ Define f(2) so that f(x) = (x^2 - 4) / (x - 2) for x 2 becomes continuous at x = 2.

- Hint: Continuity requires the function value at x = 2 to match the limit of the nearby expression.
- Step 1: Factor the numerator and simplify the expression for x not equal to 2.
- Step 2: Use the simplified expression to compute the limit as x approaches 2.
- Step 3: Set the missing function value equal to that limit.
- Checkpoint: Set f(2) = 4

Chapter homework

@@TOKEN_0@@ Core algebraic manipulations, derivative mechanics, and interpretation of slope.

1. Compute lim x->9 (sqrt(x) - 3) / (x - 9) and justify each algebraic step.
2. Use the derivative definition to differentiate f(x) = 3x^2 - 2x.
3. Find the tangent line to y = x^3 - 4x at x = 2.
4. Determine where f(x) = x^4 - 4x^2 is increasing and where it is decreasing.

Answers for these homework problems appear in the back-of-book answer key.

Chapter summary and study notes

- Recognize when a graph has a mismatch between function value and limiting value.
- Use factoring, rationalization, and common denominators to remove zero-over-zero forms.
- Describe continuity on intervals and at endpoints with correct language.

Study tips

- Substitute first. If nothing breaks, stop there.
- When you get zero over zero, look for factoring, conjugates, or a common denominator before trying anything exotic.
- Check left-hand and right-hand behavior separately whenever a graph or piecewise rule changes at the target point.

Common traps

- Treating an indeterminate form as the answer instead of as a signal to simplify.
- Cancelling terms instead of factors.
- Confusing "the function value exists" with "the limit exists."

Family-level errors to watch for

- Starting algebra before identifying the governing definition or theorem.
- Dropping notation, units, or sign conventions in the middle of a calculation.
- Treating a symbolic answer as finished without interpreting what it means.

Chapter 2

Chapter 2 Derivative definition and derivative rules

Chapter purpose

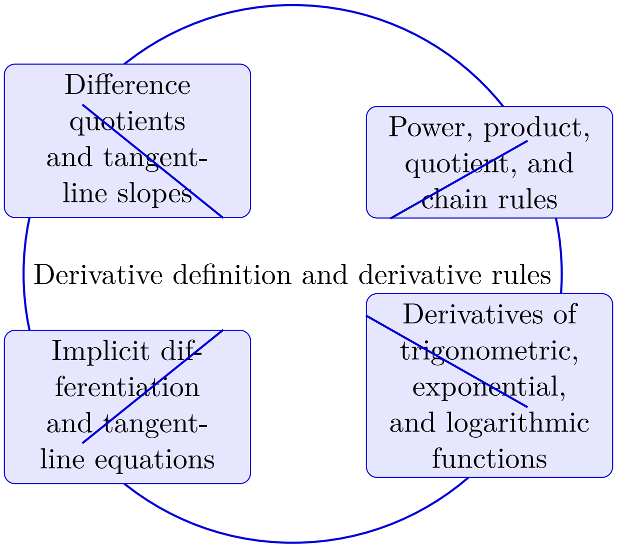
This lesson develops the derivative from first principles and then transitions into a practical rule system. Students move from secant lines to tangent lines, from average velocity to instantaneous velocity, and from the limit definition to product, quotient, and chain rules. Emphasis stays on meaning: every derivative answer must be interpreted in the context of change.

This chapter sits in the middle of Calculus I. It develops Difference quotients and tangent-line slopes, Power, product, quotient, and chain rules, Derivatives of trigonometric, exponential, and logarithmic functions, and Implicit differentiation and tangent-line equations so that the student can move from explanation to execution without losing the thread of the course.

The central habit in this chapter is to move across words, graphs, formulas, and worked algebra without losing meaning. A correct answer is not enough on its own; the student should be able to explain why the setup is valid and how the result fits the larger mathematical structure of the course.

Core ideas

- Difference quotients and tangent-line slopes
- Power, product, quotient, and chain rules
- Derivatives of trigonometric, exponential, and logarithmic functions
- Implicit differentiation and tangent-line equations



How to think through this chapter

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When working this chapter, keep the following question active: @@TOKEN_0@@ A good student answer should connect setup, assumptions, and conclusion instead of only chasing a final number or sentence.

Derivatives measure how a quantity responds when its input changes. The computational rules matter, but the real goal is to see each derivative as a statement about sensitivity, steepness, and local prediction.

From secant line to tangent line

The derivative begins with a geometric tension. Two points determine a secant line, but one point does not determine a tangent slope by itself. The only honest way to talk about the slope at one point is to compare nearby points and then ask what happens as the gap shrinks. That is why the derivative definition looks like an average rate of change with a limit attached.

Students who remember this origin make fewer mistakes later. Product rule, chain rule, and implicit differentiation all look less arbitrary when the derivative is understood as a local rate rather than just a symbol-pushing operation.

Why the chain rule is the most important rule in the course

Many expressions in engineering are layered. Temperature depends on time, pressure depends on temperature, and some final design quantity depends on pressure. Composite functions are everywhere, so the derivative must respect the fact that one change travels through another. The chain rule is calculus learning to follow influence through layers.

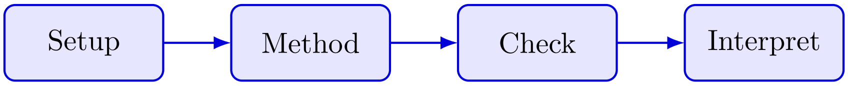
A simple trick helps here: name the layers in words before differentiating. If a student can say "the outer function is a power and the inner function is a quadratic," the derivative usually becomes much easier to organize.

Derivative answers should still sound like English

A derivative is not complete when the symbols are finished. The student should still be able to say what the sign means, what the magnitude suggests, and what the units say about the underlying situation. Positive means increasing. Negative means decreasing. Large magnitude means high sensitivity. Units explain what is changing with respect to what.

That translation back into words is not decoration. It is how students notice nonsense answers. If the derivative suggests a length is changing in square feet per second, the units alone tell you to slow down and check the setup.

Worked example



@@TOKEN_0@@ Differentiate y = (x^2 + 1)^5 and interpret the structure of the answer.

1. The outer function is u^5 and the inner function is u = x^2 + 1.
2. Differentiate the outer layer first: 5(x^2 + 1)^4.
3. Multiply by the derivative of the inner layer, 2x.
4. The derivative is y' = 10x(x^2 + 1)^4, which shows both growth of the outer power and the rate of the inner quadratic.

Read this example twice: once for the flow of ideas and once for the technical structure of the solution.

Worked-through guided example

@@TOKEN_0@@ Differentiate y = (3x^2 - 1)^4.

1. Treat u = 3x^2 - 1 so the expression becomes u^4.
2. Differentiate the outer layer to get 4(3x^2 - 1)^3.
3. Multiply by the derivative of the inner layer, which is 6x.

The chain rule gives y’ = 4(3x^2 - 1)^3 ů 6x = 24x(3x^2 - 1)^3.

Instructor commentary

Students should annotate this chapter for structure, not just facts. Mark where the argument changes direction, where the method requires a hidden assumption, and where the conclusion becomes more general than the worked example. If the chapter feels easy while you are reading it but difficult when you close the page, you have not yet converted recognition into mastery.

The most effective study pattern is read, annotate, rebuild the worked example without looking, and then solve several short-to-long problems in one sitting so the idea becomes automatic.

Practice while you read

Practice Set: Derivative mechanics

Strengthen rule selection, composite differentiation, and tangent-line interpretation.

@@TOKEN_0@@ Differentiate y = (3x^2 - 1)^4.

- Hint: This is a composite function. Separate the outer power from the inner quadratic before differentiating.
- Step 1: Treat u = 3x^2 - 1 so the expression becomes u^4.
- Step 2: Differentiate the outer layer to get 4(3x^2 - 1)^3.
- Step 3: Multiply by the derivative of the inner layer, which is 6x.
- Checkpoint: y’ = 24x(3x^2 - 1)^3

@@TOKEN_0@@ Find the equation of the tangent line to y = x^2 + 2x at x = 1.

- Hint: A tangent line needs two pieces: the point on the curve and the derivative value at that point.
- Step 1: Evaluate the function at x = 1 to get the point on the curve.
- Step 2: Differentiate y = x^2 + 2x and then evaluate the derivative at x = 1.
- Step 3: Use point-slope form with the point and slope you found.
- Checkpoint: Tangent line: y = 4x - 1

Chapter homework

@@TOKEN_0@@ Core algebraic manipulations, derivative mechanics, and interpretation of slope.

1. Compute lim x->9 (sqrt(x) - 3) / (x - 9) and justify each algebraic step.
2. Use the derivative definition to differentiate f(x) = 3x^2 - 2x.
3. Find the tangent line to y = x^3 - 4x at x = 2.
4. Determine where f(x) = x^4 - 4x^2 is increasing and where it is decreasing.

Answers for these homework problems appear in the back-of-book answer key.

Chapter summary and study notes

- Set up the derivative definition correctly before using shortcuts.
- Differentiate composite functions without dropping inner derivatives.
- Translate derivative notation among f prime, dy/dx, and operator notation.

Study tips

- Rewrite radicals and reciprocals as powers before differentiating.
- Mark outer and inner layers explicitly on any composite function.
- After every derivative, ask what the sign and units mean in context.

Common traps

- Dropping the derivative of the inside function in chain-rule problems.
- Forgetting that implicit differentiation requires differentiating every term with respect to x.
- Treating notation changes such as f prime and dy/dx as different ideas instead of different labels for the same rate.

Family-level errors to watch for

- Starting algebra before identifying the governing definition or theorem.
- Dropping notation, units, or sign conventions in the middle of a calculation.
- Treating a symbolic answer as finished without interpreting what it means.

Chapter 3

Chapter 3 Applications of derivatives

Chapter purpose

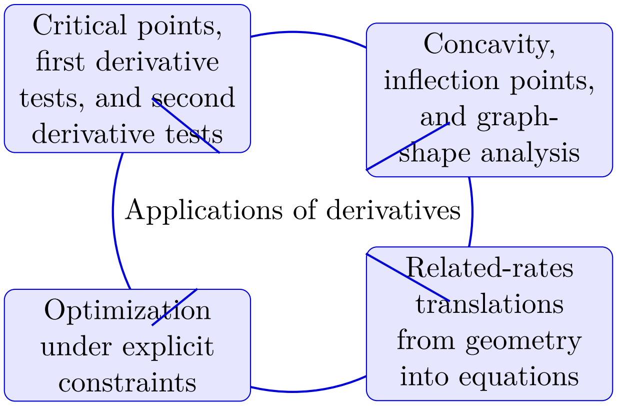
Once students can compute derivatives, they use them to reason about behavior. The focus here is monotonicity, concavity, extrema, linear approximation, Newton's method, related rates, and classical optimization. The hard part is selecting the right variables and constraints before any calculus starts, so the lesson repeatedly asks students to model with care.

This chapter sits in the middle of Calculus I. It develops Critical points, first derivative tests, and second derivative tests, Concavity, inflection points, and graph-shape analysis, Related-rates translations from geometry into equations, and Optimization under explicit constraints so that the student can move from explanation to execution without losing the thread of the course.

The central habit in this chapter is to move across words, graphs, formulas, and worked algebra without losing meaning. A correct answer is not enough on its own; the student should be able to explain why the setup is valid and how the result fits the larger mathematical structure of the course.

Core ideas

- Critical points, first derivative tests, and second derivative tests
- Concavity, inflection points, and graph-shape analysis
- Related-rates translations from geometry into equations
- Optimization under explicit constraints



How to think through this chapter

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When working this chapter, keep the following question active: @@TOKEN_0@@ A good student answer should connect setup, assumptions, and conclusion instead of only chasing a final number or sentence.

Derivative applications are where students discover whether they understand calculus structurally or only mechanically. The hard part is usually not the derivative itself. The hard part is deciding what quantity to optimize, what stays fixed, and what story the sign of the derivative is telling.

Applications are modeling problems first

Many students lose application problems by starting the derivative too early. In related rates and optimization, calculus enters only after the quantities are named correctly and the constraint has been written down. A badly chosen variable or missing relationship will defeat perfect differentiation.

The better habit is to write a one-sentence inventory before computing: what changes, what does not change, and what quantity the problem actually cares about. That sentence often reveals the entire route to the solution.

Critical points are candidates, not guarantees

A derivative equal to zero does not automatically mean maximum or minimum. It means the graph has stopped tilting upward or downward at that instant, so the point deserves inspection. The point might be a high point, a low point, or neither. That is why first-derivative tests and second-derivative tests matter.

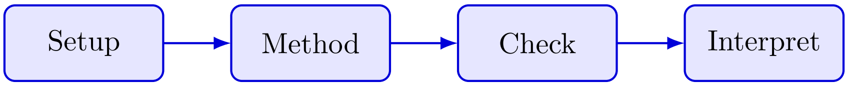
This is one of the first places where calculus trains judgment. Students stop worshipping an equation like f prime equals zero and instead use it as evidence in a broader argument about function behavior.

Approximation is one of calculus strongest powers

Linear approximation and Newton methods show that derivatives are not only descriptive. They are predictive. Near a point, a complicated function can often be replaced by a far simpler local model. That local model is rarely perfect, but it is often good enough to guide design, estimate change, or begin an iterative solve.

That engineering mindset is worth naming early: exact solutions are valuable, but controlled approximation is often more useful in practice. Students should learn to respect both.

Worked example



@@TOKEN_0@@ A rectangle has perimeter 40. Find the dimensions that maximize area.

1. Let the sides be x and y. The perimeter constraint is 2x + 2y = 40, so y = 20 - x.
2. Area is A(x) = x(20 - x) = 20x - x^2.
3. Differentiate: A'(x) = 20 - 2x. Set equal to zero to get x = 10.
4. Then y = 10, so the maximum-area rectangle is a square.

Read this example twice: once for the flow of ideas and once for the technical structure of the solution.

Worked-through guided example

@@TOKEN_0@@ A rectangle has area 48. Find the dimensions with minimum perimeter.

1. Let the sides be x and y with xy = 48, so y = 48 / x.
2. Write perimeter as P(x) = 2x + 96 / x.
3. Differentiate, set the derivative equal to zero, and solve for x.

P'(x) = 2 - 96/x^2. Setting this equal to zero gives x^2 = 48, so x = 43. Then y = 48/x = 43, so the minimum-perimeter rectangle is a square.

Instructor commentary

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The most effective study pattern is read, annotate, rebuild the worked example without looking, and then solve several short-to-long problems in one sitting so the idea becomes automatic.

Practice while you read

Practice Set: Derivative applications

Model optimization and related-rates questions without jumping to differentiation too early.

@@TOKEN_0@@ A rectangle has area 48. Find the dimensions with minimum perimeter.

- Hint: Write perimeter in one variable by using the fixed-area constraint first.
- Step 1: Let the sides be x and y with xy = 48, so y = 48 / x.
- Step 2: Write perimeter as P(x) = 2x + 96 / x.
- Step 3: Differentiate, set the derivative equal to zero, and solve for x.
- Checkpoint: Minimum-perimeter rectangle: 43 by 43

@@TOKEN_0@@ A spherical balloon is inflated so its radius grows at 0.5 cm/s. How fast is the volume changing when the radius is 6 cm?

- Hint: Use the volume formula V = (4/3)r^3 and differentiate with respect to time.
- Step 1: Differentiate V = (4/3)r^3 implicitly with respect to time t.
- Step 2: Substitute r = 6 and dr/dt = 0.5 into the derivative formula.
- Step 3: Simplify the result and keep the cubic-centimeters-per-second units.
- Checkpoint: dV/dt = 72 cm^3/s

Chapter homework

@@TOKEN_0@@ Optimization, related rates, accumulation, and interpretation of definite integrals.

1. A ladder 13 ft long slides down a wall. When the base is 5 ft from the wall and moving at 2 ft/s, how fast is the top sliding?

2. Use four right-endpoint rectangles to approximate integral from 0 to 2 of x^2 dx.
3. Find the dimensions of the open-top box with square base that has volume 32 cubic units and minimum surface area.
4. Evaluate integral from -2 to 3 of (3x - 1) dx and interpret the sign of the result.

Answers for these homework problems appear in the back-of-book answer key.

Chapter summary and study notes

- Separate what is changing from what is fixed in a word problem.
- Use a constraint to reduce an optimization problem to one variable.
- Interpret a derivative sign chart instead of treating it as a ritual.

Study tips

- In optimization, eliminate variables before differentiating.
- Draw a quick sketch even if the problem does not ask for one.
- When testing extrema, interpret intervals, not just isolated points.

Common traps

- Differentiating before writing the governing constraint.
- Assuming every critical point is an extremum.
- Using formulas mechanically without checking whether the result matches the geometry of the problem.

Family-level errors to watch for

- Starting algebra before identifying the governing definition or theorem.
- Dropping notation, units, or sign conventions in the middle of a calculation.
- Treating a symbolic answer as finished without interpreting what it means.

Chapter 4

Chapter 4 Antiderivatives, Riemann sums, and the Fundamental Theorem

Chapter purpose

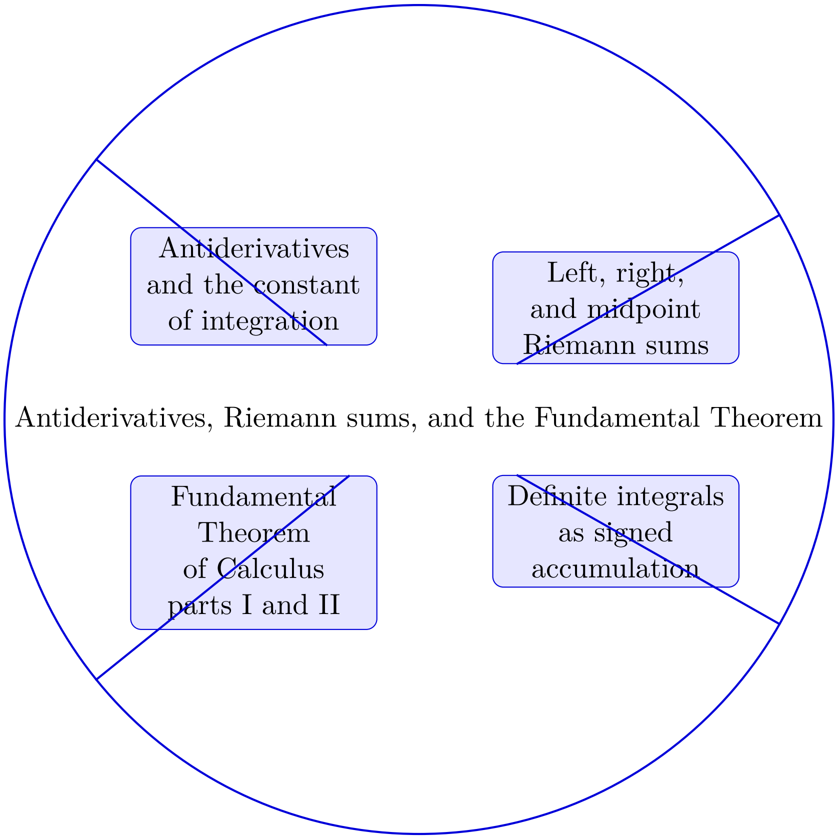
The final Calculus I lesson reframes area as accumulated change. Students approximate accumulation with Riemann sums, then formalize it with the definite integral. The course closes by connecting differentiation and integration through the Fundamental Theorem of Calculus so students can move cleanly into a more advanced integration course.

This chapter sits at the end of Calculus I. It develops Antiderivatives and the constant of integration, Left, right, and midpoint Riemann sums, Definite integrals as signed accumulation, and Fundamental Theorem of Calculus parts I and II so that the student can move from explanation to execution without losing the thread of the course.

The central habit in this chapter is to move across words, graphs, formulas, and worked algebra without losing meaning. A correct answer is not enough on its own; the student should be able to explain why the setup is valid and how the result fits the larger mathematical structure of the course.

Core ideas

- Antiderivatives and the constant of integration
- Left, right, and midpoint Riemann sums
- Definite integrals as signed accumulation
- Fundamental Theorem of Calculus parts I and II



How to think through this chapter

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When working this chapter, keep the following question active: @@TOKEN_0@@ A good student answer should connect setup, assumptions, and conclusion instead of only chasing a final number or sentence.

The integral enters Calculus I as the answer to a different question than the derivative. Instead of asking how fast something is changing right now, it asks how many tiny changes have accumulated over an interval.

Accumulation is the big idea

Riemann sums teach the student to build a large total out of small pieces. Each rectangle is a crude local contribution. The sum is an organized guess about a total amount. The definite integral is what happens when the pieces become finer and the approximation settles to a stable value.

That point of view is much more powerful than memorizing "integral equals area." Area is one common picture, but accumulation also covers displacement from velocity, mass from density, charge from current, and many other engineering quantities.

Antiderivatives are useful because differentiation and accumulation are linked

The Fundamental Theorem of Calculus is one of the central surprises of mathematics. It says the process of accumulating change and the process of measuring instantaneous change are deeply connected. That is why a difficult accumulation problem can often be evaluated by finding an antiderivative.

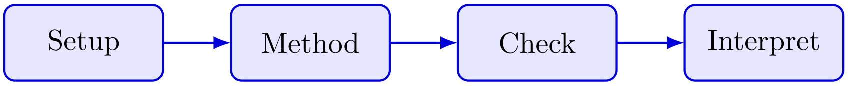
Students should not rush past that theorem as a computational convenience only. It is the bridge that turns the first half of the course into one coherent story instead of two unrelated chapters.

Signed area is a feature, not a flaw

A definite integral can be negative because it records net change. Regions below the axis count negatively. This is not the integral making a mistake. It is the integral being honest about direction and sign.

Whenever the problem asks for total accumulated quantity rather than net effect, students must pause and decide whether to split the interval or use absolute value. That choice is conceptual, not cosmetic.

Worked example



@@TOKEN_0@@ Evaluate integral from 1 to 3 of (2x + 1) dx.

1. Find an antiderivative: x^2 + x.
2. Evaluate at the upper limit: 3^2 + 3 = 12.
3. Evaluate at the lower limit: 1^2 + 1 = 2.
4. Subtract to obtain 12 - 2 = 10.

Read this example twice: once for the flow of ideas and once for the technical structure of the solution.

Worked-through guided example

@@TOKEN_0@@ Evaluate integral from 0 to 2 of (3x^2 - 4x + 1) dx.

1. Integrate term by term to build an antiderivative.
2. Substitute x = 2 into the antiderivative.
3. Subtract the value at x = 0.

An antiderivative is x^3 - 2x^2 + x. Evaluating from 0 to 2 gives (8 - 8 + 2) - 0 = 2.

Instructor commentary

Students should annotate this chapter for structure, not just facts. Mark where the argument changes direction, where the method requires a hidden assumption, and where the conclusion becomes more general than the worked example. If the chapter feels easy while you are reading it but difficult when you close the page, you have not yet converted recognition into mastery.

The most effective study pattern is read, annotate, rebuild the worked example without looking, and then solve several short-to-long problems in one sitting so the idea becomes automatic.

Practice while you read

Practice Set: Antiderivatives and accumulation

Reinforce antiderivatives, definite integrals, and the difference between net change and total change.

@@TOKEN_0@@ Evaluate integral from 0 to 2 of (3x^2 - 4x + 1) dx.

- Hint: Find an antiderivative first, then evaluate it at the upper and lower bounds.
- Step 1: Integrate term by term to build an antiderivative.
- Step 2: Substitute x = 2 into the antiderivative.
- Step 3: Subtract the value at x = 0.
- Checkpoint: Integral value: 2

@@TOKEN_0@@ Use a left-endpoint Riemann sum with 4 rectangles to approximate integral from 0 to 4 of x + 1 dx.

- Hint: Compute x first, then list the left endpoints of each subinterval.
- Step 1: The interval length is 4 and there are 4 rectangles, so x = 1.

- Step 2: The left endpoints are 0, 1, 2, and 3. Evaluate f(x) = x + 1 at each one.
- Step 3: Add those function values and multiply by x.
- Checkpoint: Left-endpoint approximation: 10

Chapter homework

@@TOKEN_0@@ Optimization, related rates, accumulation, and interpretation of definite integrals.

1. A ladder 13 ft long slides down a wall. When the base is 5 ft from the wall and moving at 2 ft/s, how fast is the top sliding?
2. Use four right-endpoint rectangles to approximate integral from 0 to 2 of x^2 dx.
3. Find the dimensions of the open-top box with square base that has volume 32 cubic units and minimum surface area.
4. Evaluate integral from -2 to 3 of (3x - 1) dx and interpret the sign of the result.

Answers for these homework problems appear in the back-of-book answer key.

Chapter summary and study notes

- Write a sigma expression for a Riemann sum from an interval partition.
- Explain the difference between net area and total area.
- Use the Fundamental Theorem to evaluate definite integrals efficiently.

Study tips

- Before evaluating an integral, ask whether the quantity should be net change or total amount.
- In Riemann sums, identify delta x and the sample point rule before writing sigma notation.
- Differentiate your antiderivative mentally to confirm it matches the integrand.

Common traps

- Forgetting the constant of integration on indefinite integrals.
- Treating signed area and total area as the same quantity.
- Applying the Fundamental Theorem without checking whether the antiderivative was found correctly.

Family-level errors to watch for

- Starting algebra before identifying the governing definition or theorem.
- Dropping notation, units, or sign conventions in the middle of a calculation.
- Treating a symbolic answer as finished without interpreting what it means.

Chapter 5

Quiz review and official exam preparation

Homework structure

- Homework Set 1: Limits and derivatives: 4 graded problems attached to chapter 1.
- Homework Set 2: Applications and the first integrals: 4 graded problems attached to chapter 2.

Quiz structure

- Quiz 1: Limits and derivative fundamentals: 4 questions, timed, and single-attempt in the live course. Quiz 1 should be taken only after you can solve the chapter homework without outside prompts.
- Quiz 2: Applications and the Fundamental Theorem: 4 questions, timed, and single-attempt in the live course. Quiz 2 should be taken only after you can solve the chapter homework without outside prompts.

Official mastery exam

- Calculus I cumulative mastery exam: 5 major questions, High rigor, first official attempt locks the course grade.

Calculus I cumulative mastery exam preparation checklist

- Be able to justify every algebraic simplification in a limit.
- Memorize core derivative rules but also know when the definition is the right tool.
- Practice translating geometry and motion language into equations before differentiating.

- Review both antiderivatives and definite integrals so you know when each form is appropriate.

How to use this book before assessment

- Read the relevant chapter and rebuild both worked examples without looking.
- Solve the guided practice in the chapter before attempting the graded homework.
- Check your chapter-homework answers only after you complete a full written attempt.
- Review the quiz answer key after each chapter block and classify your errors by concept, setup, algebra, or interpretation.
- Before the official exam, revisit the chapter purposes, homework corrections, and answer-key notes rather than rereading formulas only.

Chapter 6

Course vocabulary index

- @@TOKEN_0@@: treat this as a working term in the course. You should be able to define it, recognize where it appears, and use it correctly in a solution or explanation.
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Chapter 7

Back-of-book answers and solution outlines

Guided practice answer key

Chapter 1: Functions, limits, and continuity

@@TOKEN_0@@

1. Evaluate lim x->3 (x^2 - 9) / (x - 3).

- Checkpoint answer: Final answer: 6 - Solution note: The original form is indeterminate, but factoring removes the zero-over-zero obstacle. After cancellation the limit is lim x->3 (x + 3) = 6.

1. Define f(2) so that f(x) = (x^2 - 4) / (x - 2) for x 2 becomes continuous at x = 2.

- Checkpoint answer: Set f(2) = 4 - Solution note: For x 2, the function simplifies to x + 2. The limit as x approaches 2 is 4, so continuity requires f(2) = 4.

Chapter 2: Derivative definition and derivative rules

@@TOKEN_0@@

1. Differentiate y = (3x^2 - 1)^4.

- Checkpoint answer: y' = 24x(3x^2 - 1)^3 - Solution note: The chain rule gives y' = 4(3x^2 - 1)^3 ů 6x = 24x(3x^2 - 1)^3.

1. Find the equation of the tangent line to y = x^2 + 2x at x = 1.

- Checkpoint answer: Tangent line: y = 4x - 1 - Solution note: The point is (1, 3). The derivative is 2x + 2, so the slope at x = 1 is 4. Point-slope form gives y - 3 = 4(x - 1), or y = 4x - 1.

Chapter 3: Applications of derivatives

@@TOKEN_0@@

1. A rectangle has area 48. Find the dimensions with minimum perimeter.

- Checkpoint answer: Minimum-perimeter rectangle: 43 by 43 - Solution note: P'(x) = 2 - 96/x^2. Setting this equal to zero gives x^2 = 48, so x = 43. Then y = 48/x = 43, so the minimum-perimeter rectangle is a square.

1. A spherical balloon is inflated so its radius grows at 0.5 cm/s. How fast is the volume changing when the radius is 6 cm?

- Checkpoint answer: dV/dt = 72 cm^3/s - Solution note: Differentiating gives dV/dt = 4r^2 dr/dt. At r = 6 and dr/dt = 0.5, the rate is 4(36)(0.5) = 72 cm^3/s.

Chapter 4: Antiderivatives, Riemann sums, and the Fundamental Theorem

@@TOKEN_0@@

1. Evaluate integral from 0 to 2 of (3x^2 - 4x + 1) dx.

- Checkpoint answer: Integral value: 2 - Solution note: An antiderivative is x^3 - 2x^2 + x. Evaluating from 0 to 2 gives (8 - 8 + 2) - 0 = 2.

1. Use a left-endpoint Riemann sum with 4 rectangles to approximate integral from 0 to 4 of x + 1 dx.

- Checkpoint answer: Left-endpoint approximation: 10 - Solution note: The function values are 1, 2, 3, and 4. Their sum is 10, and since x = 1 the approximation is 10.

Homework answer key

Homework Set 1: Limits and derivatives

1. Compute lim x->9 (sqrt(x) - 3) / (x - 9) and justify each algebraic step.

- Answer / solution summary: Multiply by the conjugate to get 1 / (sqrt(x) + 3); the limit is 1/6.

1. Use the derivative definition to differentiate f(x) = 3x^2 - 2x.

- Answer / solution summary: Expand f(x+h), cancel matching terms, divide by h, and let h approach 0 to get f'(x) = 6x - 2.

1. Find the tangent line to y = x^3 - 4x at x = 2.

- Answer / solution summary: f(2) = 0 and f'(x) = 3x^2 - 4, so the slope is 8. The line is y = 8(x - 2).

1. Determine where f(x) = x^4 - 4x^2 is increasing and where it is decreasing.

- Answer / solution summary: f'(x) = 4x(x^2 - 2). Use critical points x = -sqrt(2), 0, sqrt(2) to build the sign chart.

Homework Set 2: Applications and the first integrals

1. A ladder 13 ft long slides down a wall. When the base is 5 ft from the wall and moving at 2 ft/s, how fast is the top sliding?

- Answer / solution summary: At x = 5, y = 12. Differentiate to get 2x dx/dt + 2y dy/dt = 0, so dy/dt = -(x/y) dx/dt = -5/6 ft/s.

1. Use four right-endpoint rectangles to approximate integral from 0 to 2 of x^2 dx.

- Answer / solution summary: Compute 0.5[(0.5)^2 + 1^2 + (1.5)^2 + 2^2] = 3.75.

1. Find the dimensions of the open-top box with square base that has volume 32 cubic units and minimum surface area.

- Answer / solution summary: Volume gives h = 32 / x^2. Surface area is x^2 + 4xh = x^2 + 128/x. Differentiate and solve x = 4, h = 2.

1. Evaluate integral from -2 to 3 of (3x - 1) dx and interpret the sign of the result.

- Answer / solution summary: An antiderivative is 1.5x^2 - x. The value is 7.5, meaning net signed accumulation is positive overall.

Quiz answer key

Quiz 1: Limits and derivative fundamentals

1. If lim x->a f(x) exists, which statement must also be true?

- Answer key: The left and right limits at a agree. A two-sided limit exists exactly when the one-sided limits exist and match.

1. Find f'(2) if f(x) = x^3 - 4x.

- Answer key: Accepted answer(s): 8, 8.0. f'(x) = 3x^2 - 4, so f'(2) = 12 - 4 = 8.

1. The derivative of ln(x^2 + 1) is:

- Answer key: 2x / (x^2 + 1). Use the chain rule: derivative of ln(u) is u'/u.

1. If f' changes from positive to negative at c, then f has a:

- Answer key: Local maximum at c. Positive to negative means the function rises and then falls, giving a local maximum.

Quiz 2: Applications and the Fundamental Theorem

1. Which quantity is minimized in a standard optimization setup?

- Answer key: A single objective function after substitution. Optimization succeeds only after the constraint has reduced the objective to one independent variable.

1. Evaluate d/dx of integral from 1 to x of (t^2 + 3) dt.

- Answer key: Accepted answer(s): x^2 + 3, x^2+3. By the Fundamental Theorem of Calculus, the derivative returns the integrand evaluated at x.

1. A negative definite integral always means:

- Answer key: Net signed area is below the axis overall. A definite integral measures signed accumulation, not absolute area.

1. If a ladder problem uses x^2 + y^2 = L^2, what must happen before solving for dy/dt?

- Answer key: Differentiate the entire equation with respect to time. Related-rates problems require implicit differentiation with respect to time.

Mastery exam solution outlines

Calculus I cumulative mastery exam

1. Evaluate lim x->0 [sin(5x)] / x using a correct limit argument, not only pattern recognition.

- What to show: A valid rewrite or theorem reference; Why the standard sin(u) / u limit applies - Solution outline: Rewrite as 5[sin(5x)/(5x)] and use the standard limit as 5x approaches 0. The value is 5.

1. A company models cost by C(q) = q^3 - 9q^2 + 30q + 50. Find intervals of increasing and decreasing cost and classify every critical point.

- What to show: Derivative and critical-point computation; A sign chart and classification - Solution outline: Differentiate to get C'(q) = 3q^2 - 18q + 30 = 3(q^2 - 6q + 10). The discriminant is negative, so there are no real critical points and the derivative stays positive. Cost is increasing for all real q and has no local extrema.

1. Water is poured into a conical tank with height 12 and radius 6 at 3 cubic feet per minute. How fast is the water depth rising when the depth is 4?

- What to show: Similarity relationship between radius and height; A correct volume formula in one variable - Solution outline: From similar triangles, r/h = 6/12 = 1/2 so r = h/2. Volume is (1/3)pi r^2 h = (1/3)pi (h^2 / 4)h = pi h^3 / 12. Differentiate: dV/dt = (pi/4)h^2 dh/dt. At h = 4 and dV/dt = 3, dh/dt = 3 / (4pi).

1. Use six equal subintervals and midpoint rectangles to approximate integral from 0 to 3 of e^(-x). Then state whether the approximation is an overestimate or underestimate and why.

- What to show: Correct delta x and midpoint list; A conclusion about concavity or monotonicity - Solution outline: Delta x = 0.5 with midpoints 0.25, 0.75, 1.25, 1.75, 2.25, 2.75. Compute 0.5 times the sum of e^(-midpoint). Because e^(-x) is concave up, the midpoint rule gives an underestimate.

1. Evaluate integral from 0 to 2 of (x^2 + 4x - 1) dx and explain the geometric meaning of the answer.

- What to show: A correct antiderivative; A short interpretation of signed accumulation - Solution outline: An antiderivative is x^3/3 + 2x^2 - x. The value is 26/3. The result measures net signed area, so regions below the axis would subtract if present.

Reference note

For the full bibliography behind this textbook, use @@TOKEN_0@@. The answer key in this book is Summit-authored and aligned to the live course runtime.